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Physics, Mechanics, Mathematics

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IN THE NAME OF GOD

Investigation has been made into 3 issues each studied separately under the following titles:

Topics

1- Stable Dynamics (5th dimension)
2- Dynamics Trigonometry
3- Angular Speed

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Angular speed

Abstract

In this paper , the angular speed formula has been established (proved) on any type of curve. Regarding the importance of the angular speed calculation in most of applied sciences such as dynamic mechanics , aerospace , dynamic systems and lock of a relation established in this connection , the need is felt that in order to design and optimize dynamic systems , a reasonable relation should be presented . This paper tries to prove such a relation in the easiest possible way.

Key Word : angular speed

Introduction

Angular speed on any curve : In dynamic mechanics literature , "angular speed " has been defined like this : The angle covered by a moving object in time unit.
The unit of angular speed is radiant per second (Rad / sec) .
Angular speed formula : V = R.ω
Where V = linear speed on the curve , R = radius of rotation round the rotation axis and ω = the moving object angular speed.
N this paper angular speed is studied in any type of (closed / open) curve . It is initially studied in a closed curve in which there are 2 existing characteristic geometric forms which are common : circle and ellipse .
In the open – type curve , it includes any type of cure both ends of which are not linked to each other .
Now to begin with , a closed curve is studied , firstly a circle .
Angular speed in a circle consists of 2 parts :
A .Angular speed in a circle with constant linear speed linear velocity (V) is constant in a circle and the circle radius R is also constant . Angular speed is thus constant : V = R.ω

All the angles covered in equal time units will thus be ω = ${}^{V}/_{R}$= cte (with one $\overline{\mathrm{OA}}$ link)
B. Angular speed in the circle with noncontact (variable) linear speed.
C . Angular speed in the circle with arms length more than one , in different rotational motion directions and constant & variable angular speeds.

B . Since linear velocity (v) is not constant in a circle , the circle radius (R) is however constant , angular speed will thus be noncontact . We will thus have :

R = m (circle radius in meters)

$$\omega = \frac{\mathrm{V}}{\mathrm{R}}\ {}^{\mathrm{Rad}}/_{\mathrm{sec}}$$

$$\mathrm{V} = {}^{\mathrm{m}}/_{\mathrm{sec}}\ (\text{linear velocity })$$

$$\mathrm{V} = \mathrm{R} * \omega$$

$$\mathrm{dV} = \mathrm{R} * \mathrm{d}\,\omega$$

Now regarding fig.1 it can be expressed that the moving object has moved from point A to point B in a time unit . That is , angle (α) , and hase moved from point (B) to point (C) in another time unit . That is angle (β)
Angle (α) will not equal angle (β) .
It can thus be concluded that all the angles will not be covered in equal time units, …. , in any time unit , an angle will be covered depending on the linear velocity variations .

Angular speed in an ellipse:

Given the ellipse center as the rotation axis and the moving object moves so that it holds a constant angular speed , regarding the fact that rotation radius and linear velocity are variable formula V = R.ω can be investigated :

dV = R * dω + ω.dR = 0 + ω.dR

According to Fig.2, the angles covered are equal to each other. Yet, regarding the Fig.1 it can be concluded that the linear velocity variations are proportional to the rotation radius variations.
Now, angular speed is studied in an open curve to prove the angular speed formula.

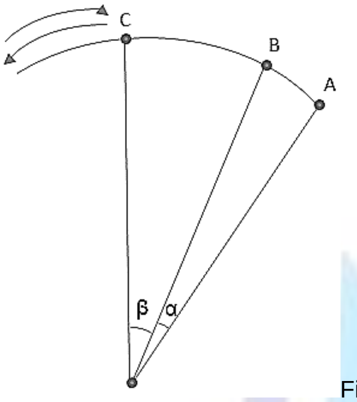

Fig. 1

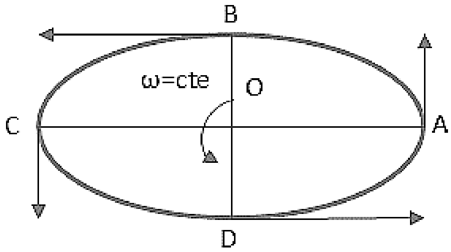

Fig. 2

In this paper, the angle the moving object covers in time unit is γ. That is, the moving object moves on the curve from point A to point B in time unit thus equals γ (Fig.3)

ω = (γ.t)

t = 1

ω = (γ.1) = (γ)

In triangle O'CH we have :

γ + β + π – α = π

γ + β = α

γ = α – β

N.B. : Angle γ may be γ = α ±β , depending on codirectional or counterdirectional speed .

$V_{xA} = {}^{m}/_{sec}$: Speed on the abscissa at point A

$V_{yA} = {}^{m}/_{sec}$: Speed on the ordinate at point B

$V_{xB} = {}^{m}/_{sec}$: Speed on the abscissa at point B

$V_{yB} = {}^{m}/_{sec}$: Speed on the ordinate at point B

Its proved formula is discussion and proof .

$$\tan\omega = \frac{V_{yA}.V_{xB} - V_{xA}.V_{yB}}{V_{xA}.V_{xB} + V_{yA}.V_{yB}}$$

discussion and proof:

$$\tan \propto = \frac{V_{yA}}{V_{xA}}$$

$$\tan \beta = \frac{V_{yB}}{V_{xB}}$$

$$\tan \gamma = \tan(\propto - \beta)$$

$$\tan \gamma = \tan \omega = \frac{\tan \propto - \tan \beta}{1 + \tan \propto . \tan \beta} = \frac{\frac{V_{yA}}{V_{xA}} - \frac{V_{yB}}{V_{xB}}}{1 + \frac{V_{yA}.V_{yB}}{V_{xA}.V_{xB}}}$$

$$\tan \omega = \frac{V_{yA}.V_{xB} - V_{xA}.V_{yB}}{V_{xA}.V_{xB} + V_{yA}.V_{yB}}$$

In Fig.3 , if the moving object moves in the next second from point B to a hypothesized point like D , the moving object angular speed in the next second will be defined , and the formula of angular speed in the next second between 2 point D and B shall apply according to the formula presented between 2 point B and A . It is noteworthy that the above mentioned formula has been proved for each time unit. For example , the moving object is at second (t_n) at a point like A on the curve , and at second (t_n + 1) at a point like B. The above mentioned proved formula thus applies between 2 point in time units (t_n)

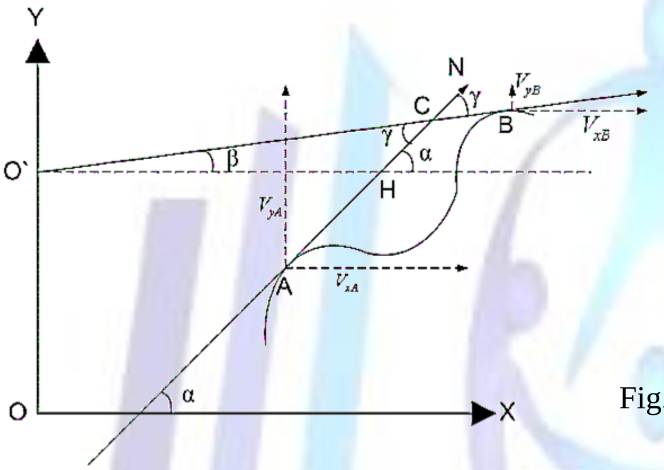

Fig. 3

and (t_n + 1). It accordingly applies between 2 points D and B at second (t_n + 1) and (t_n + 2) , etc.

Given an angular speed larger than 1, the angle covered has an average value and average angular speed is a separate option which will be described later.

Example : A moving object covers in the 1st second ($\alpha = \frac{\pi}{4}$) and in the next second ($\beta = \frac{\pi}{6}$) .

For obtaining an average angular speed , angles (α) and (β) are added and then divided by 2

$$\frac{(\propto + \beta)}{2}$$

The average angular speed will thus be obtained.

$$\frac{(\frac{\pi}{4} + \frac{\pi}{6})}{2} = \frac{20\pi}{96} = \frac{5\pi}{24}$$

It can now be concluded that angular speed is different than average angular speed , that nobody can obtain angular speed from average angular speed and that either of them is a separate option . Angular speed for 2 consecutive points , one second apart , has been proved . That is at the time between (t_n) and (t_n + 1) , angular speed will be obtained. The formula proved on an open curve also applies to a closed curve . Using an example , the accuracy of the above formula on a closed curve can be proved.

C. Examples are solved, starting with the simplest ones at constant and variable angular speeds and constant and variable relations in circles to clarify angular speed different aspects in a curve.
Ex.1. The movement of 2 arms $\{\overline{OA} = \overline{AB}\}$ at 2 codirectionally equal angular speeds (ωOA = ωAB) are considered

Angular speed : $\begin{cases} \text{constant} \\ \text{variable} \end{cases}$

Length of arm : $\begin{cases} \text{equal} \\ \text{nonequal} \end{cases}$

Direction of movement : $\begin{cases} \text{codirectional} \\ \text{counter directional} \end{cases}$

Ex.2 . Solve the 1st EM . counterdirectional relative to each other
Ex.3 . Solve Ex.1 given the following assumptions codirectional angular speed ωOA = ωAB , $\overline{OA} = \overline{2AB}$
Ex.4 . Solve Ex.1 given the following assumptions counterdirectional angular speed ωOA = ωAB , $\overline{OA} = \overline{2AB}$
Ex.5 . Solve Ex.1 given the following assumptions codirectional angular speed 2. ωOA = ωAB , $\overline{OA} = \overline{AB}$
Ex.6 . Solve Ex.1 given the following assumptions counterdirectional angular speed 2. ωOA = ωAB , $\overline{OA} = \overline{AB}$
Ex.7 . Solve Ex.1 given the following assumptions codirectional angular speed 2. ωOA = ωAB , $\overline{OA} = \overline{2AB}$
Ex.8 . Solve Ex.1 given the following assumptions codirectional angular speed 2. ωOA = ωAB , $\overline{OA} = \overline{2AB}$

Calculation of angular speed at (t) seconds in such problems , the following method can be practiced :
In Ex.3 , for calculating angular speed at t = 2 sec , using triangle $\widehat{OAB}$, length $\overline{OB}$can be calculated.
In triangle $\widehat{OAB}$, 2 sides and an angle are known .That is lengths $\overline{OA}$ and $\overline{AB}$ and angle (π-2α) are known.
Angles (γ and β) will thus also be obtained. By obtaining angle (γ) , it can be added to angle 2α. Total angles ϴ = (γ + 2α) will thus constitute angular speed covered by point B within 2 seconds . That is ωOB = (γ + 2α) . For calculating other angular speeds at different times t , the above mentioned method can be applied.
Ex.9 . Calculation of angular speed at t seconds :
In Ex.9 for calculating angular speed , we shall proceed as in Ex.3 : Firstly , using the above mentioned method in triangle $\widehat{OAB}$ applying 2 relations of $\overline{OA}$ and $\overline{AB}$, angle (γ) is obtained and then in triangle $\widehat{OBC}$, by obtaining length $\overline{OB}$ and having length $\overline{BC}$ and angle $\widehat{ABC}$, 2 angles and the other side of triangle $\widehat{OBC}$ can be obtained . By having an angle near point O (λ,θ), this angle can be added to its nearby angles , the total of which will constitute angular speed at time (t).

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Ex 1

Relations $\overline{OA} = \overline{AB}$ are given. Point (A) has angular speed $\omega OA = \frac{\pi}{18}$ relative to point (O) , and point B has angular speed $\omega AB = \frac{\pi}{18}$ relative to point (A). 2 arms $\{\overline{OA} = \overline{AB}\}$ moves towards the trigonometric circle. Please find :

1. Angular speed ωOB= ? trace its curve via drawing and calculating at different times.
2. Codirectional angular speed ωOA = ωAB , $\overline{OA} = \overline{AB}$

$$\begin{cases} \omega OA = \omega AB = \dfrac{\pi}{18}\ \mathrm{Rad/Sec} \\ t = 1\ \mathrm{sec} \\ \overline{OA} = \overline{AB} \\ \omega OB = ? \\ \alpha = \dfrac{\pi}{18}\ \mathrm{Rad} \end{cases}$$

$\overset{\triangle}{OAB} \Rightarrow (\pi - \alpha) + \gamma + \gamma = \pi \Rightarrow -\alpha + 2\gamma = 0 \Rightarrow \gamma = \frac{\alpha}{2}$

$\omega OB = \alpha + \gamma = \alpha + \frac{\alpha}{2} = \frac{3}{2}\alpha = \frac{3}{2} \cdot \frac{\pi}{18} = \frac{\pi}{12}$

t=1 sec.

…..

$\omega OB = 3\alpha + \frac{3}{2}\alpha = \frac{9\alpha}{2} = \frac{9}{2} \cdot \frac{\pi}{18} = \frac{\pi}{4}$

t=3 sec.

…..

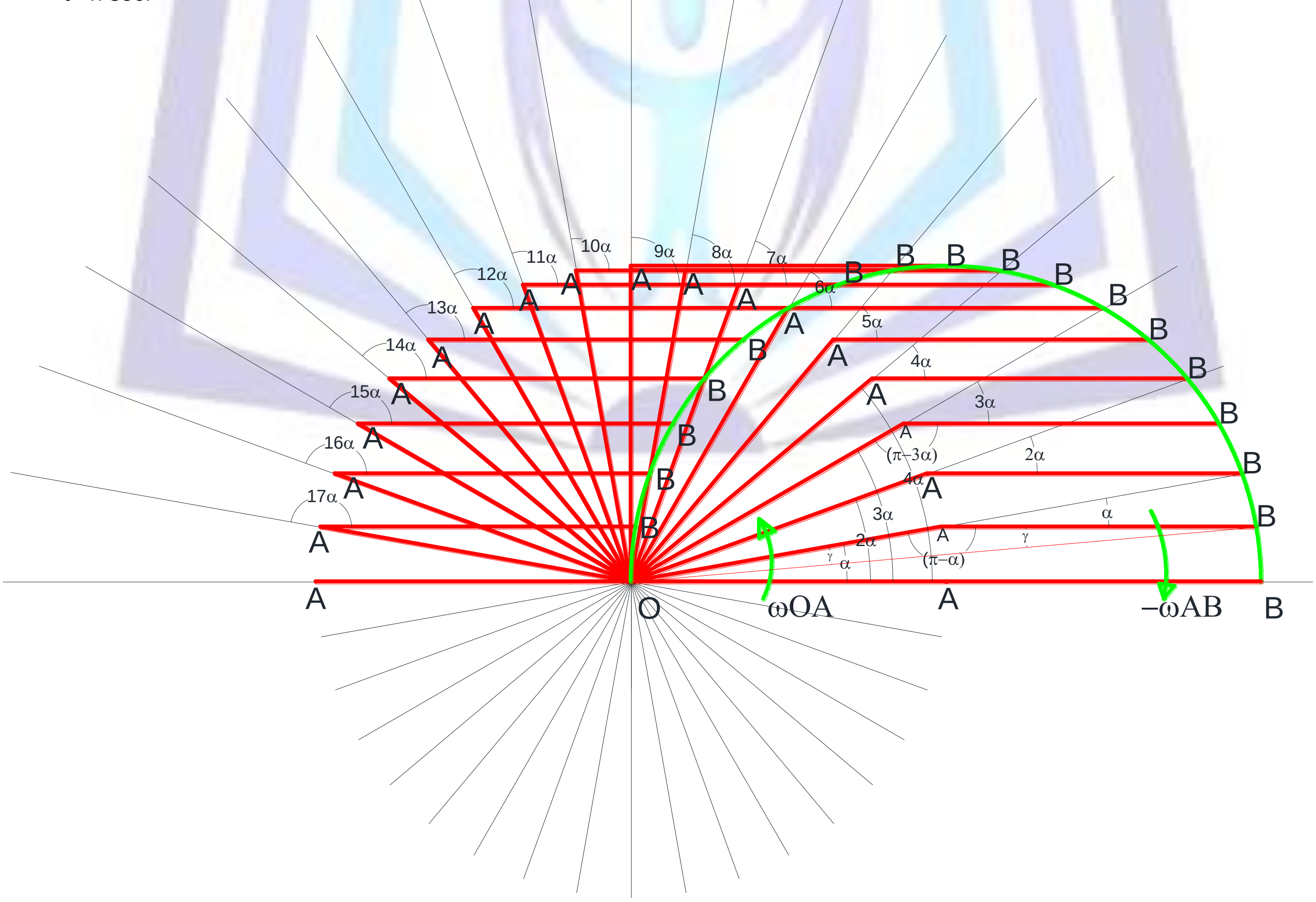
…..

$\omega OB = n\alpha + \frac{n}{2}\alpha = \frac{2n\alpha + n\alpha}{2}$

t=n sec

$\omega OB = n\alpha + \frac{n}{2}\alpha = n\alpha\left(\frac{3}{2}\right) = n \cdot \frac{\pi}{18} \cdot \frac{3}{2}$

t= n sec.

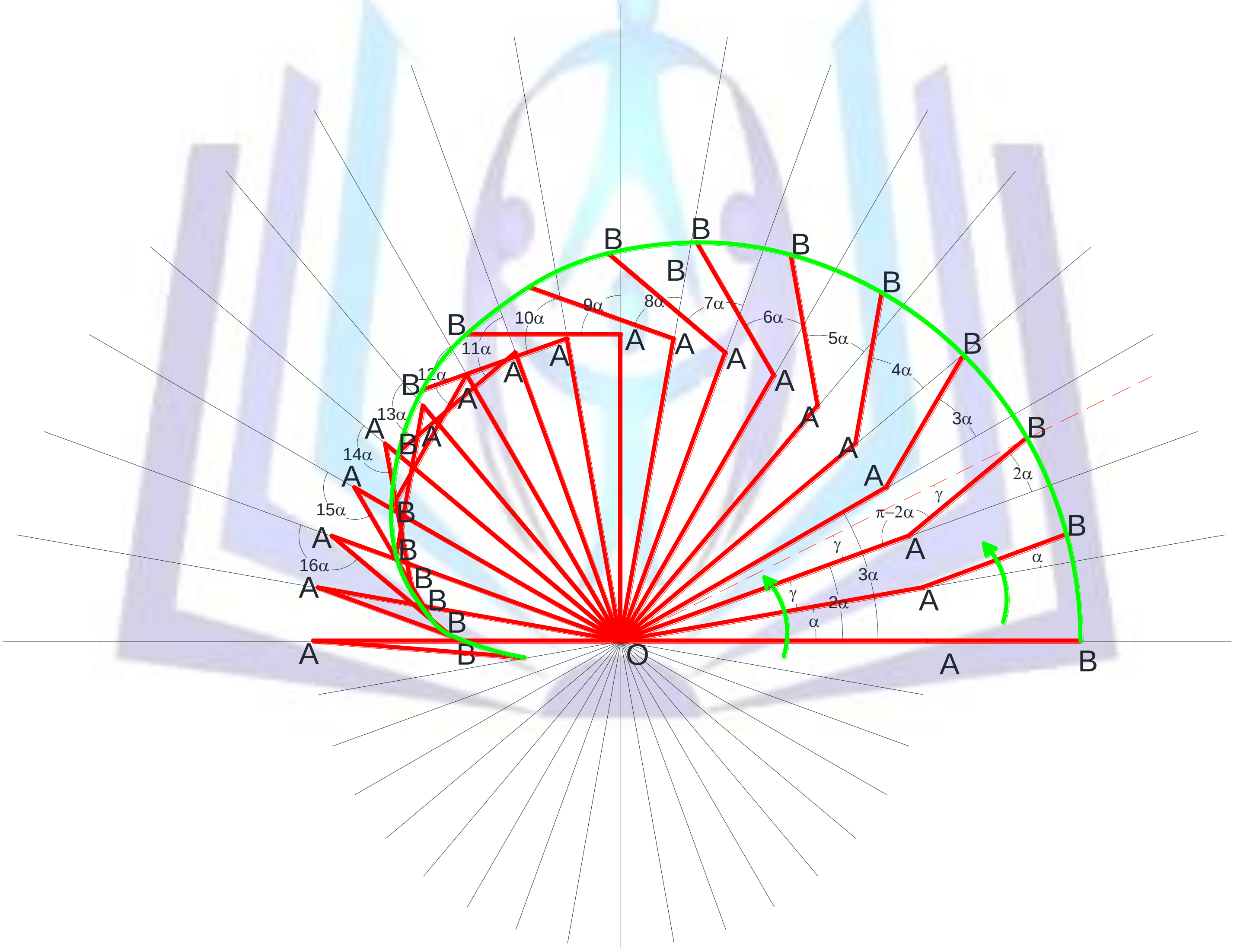


Ex 2

Codirectional angular speed

$$\begin{cases} \omega OA = \omega AB = \dfrac{\pi}{18}\,\text{Rad/Sec} \\ t = 1\,\text{sec} \\ \overline{OA} = 2.\overline{AB} \\ \omega OB = ? \\ \alpha = \dfrac{\pi}{18}\,\text{Rad} \end{cases}$$

$$\overline{OB}^2 = \overline{OA}^2 + \overline{AB}^2 - 2.\overline{OA}.\overline{AB}.\mathrm{Cos}(\pi - 2\alpha)$$



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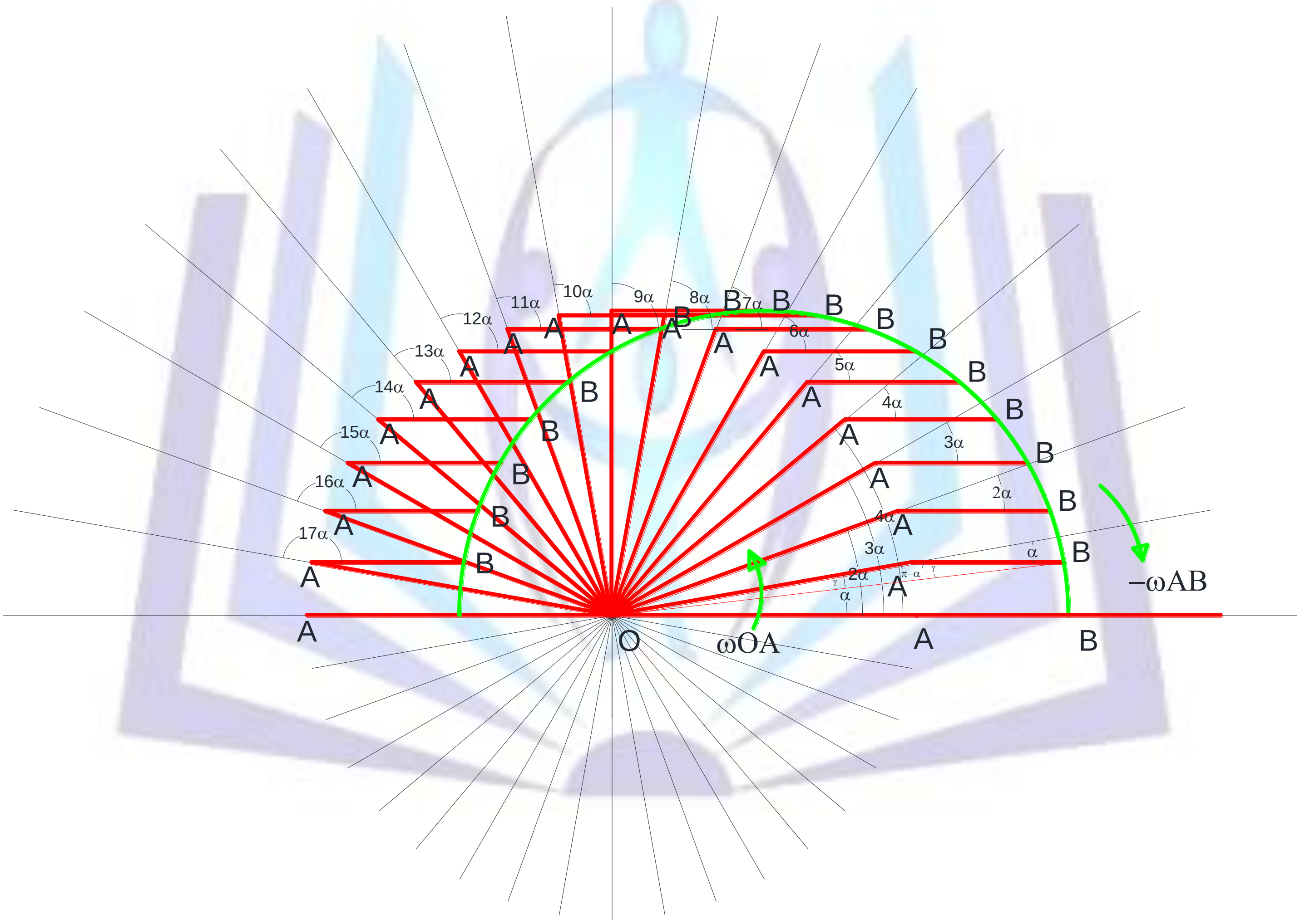
Ex 3

Counterdirectional angular speed

$$\begin{cases} \omega OA = -\omega AB = \dfrac{\frac{\pi}{18}\,\mathrm{Rad}}{\mathrm{Sec}} \\ t = 1\ \mathrm{sec} \\ \overline{OA} = 2.\overline{AB} \\ \omega OB = ? \\ \alpha = \dfrac{\pi}{18}\ \mathrm{Rad} \end{cases}$$

$$\overline{OB}^{\,2} = \overline{OA}^{\,2} + \overline{AB}^{\,2} - 2.\overline{OA}.\overline{AB}.\mathrm{Cos}(\pi - \alpha)$$



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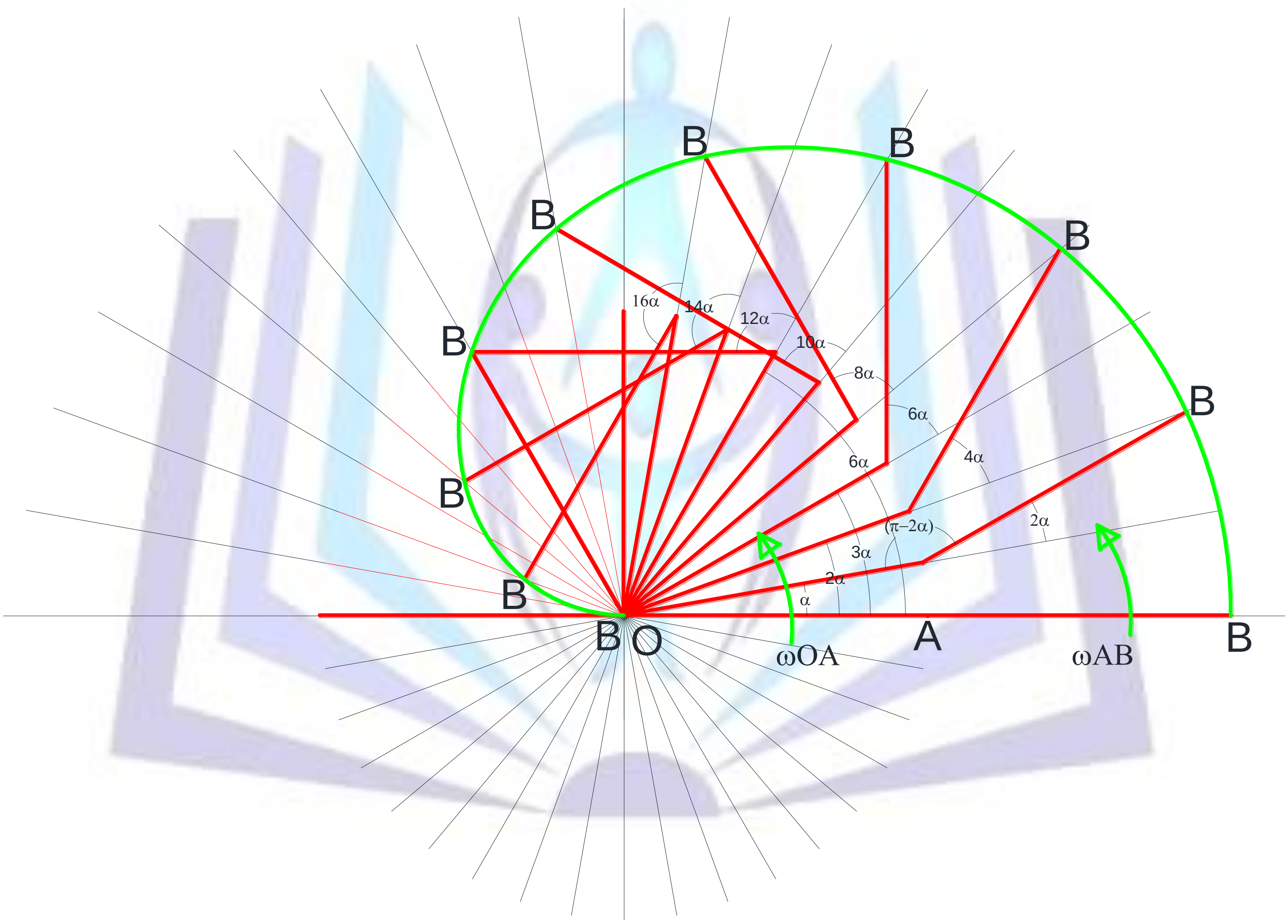
Ex 4

Codirectional angular speed

$$\begin{cases} \omega AB = 2.\omega OA \Rightarrow \omega OA = \dfrac{\pi}{18} \text{Rad/Sec} \\ t = 1 \text{ sec} \\ \overline{OA} = \overline{AB} \\ \omega OB = ? \\ \alpha = \dfrac{\pi}{18} \text{ Rad} \end{cases}$$



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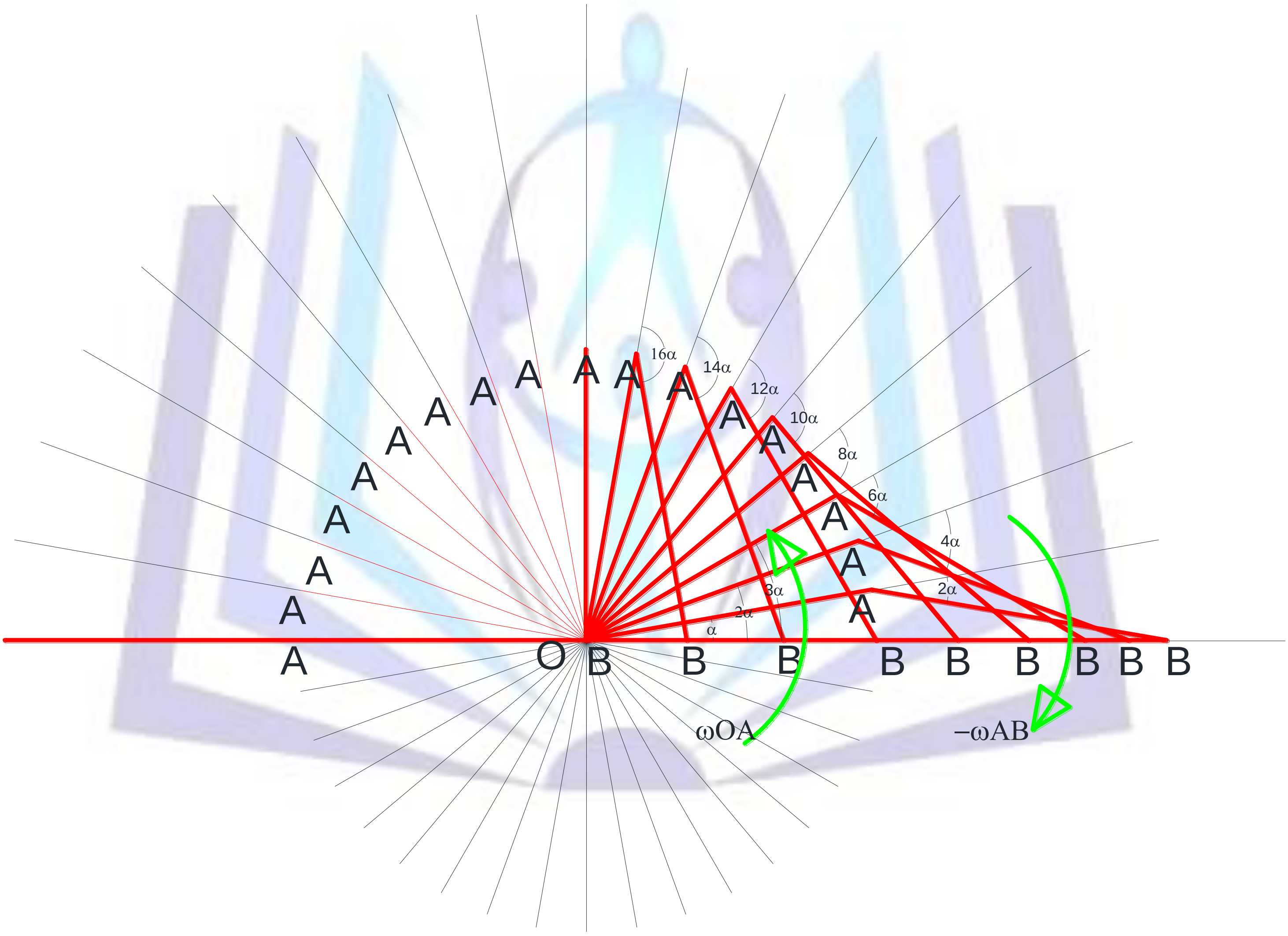
Ex 5

Counterdirectional angular speed

$$\begin{cases} -\omega AB = 2.\omega OA = \omega AO = \dfrac{\pi}{18}\mathrm{Rad/Sec} \\ t = 1\,\mathrm{sec} \\ \overline{OA} = \overline{AB} \\ \omega OB = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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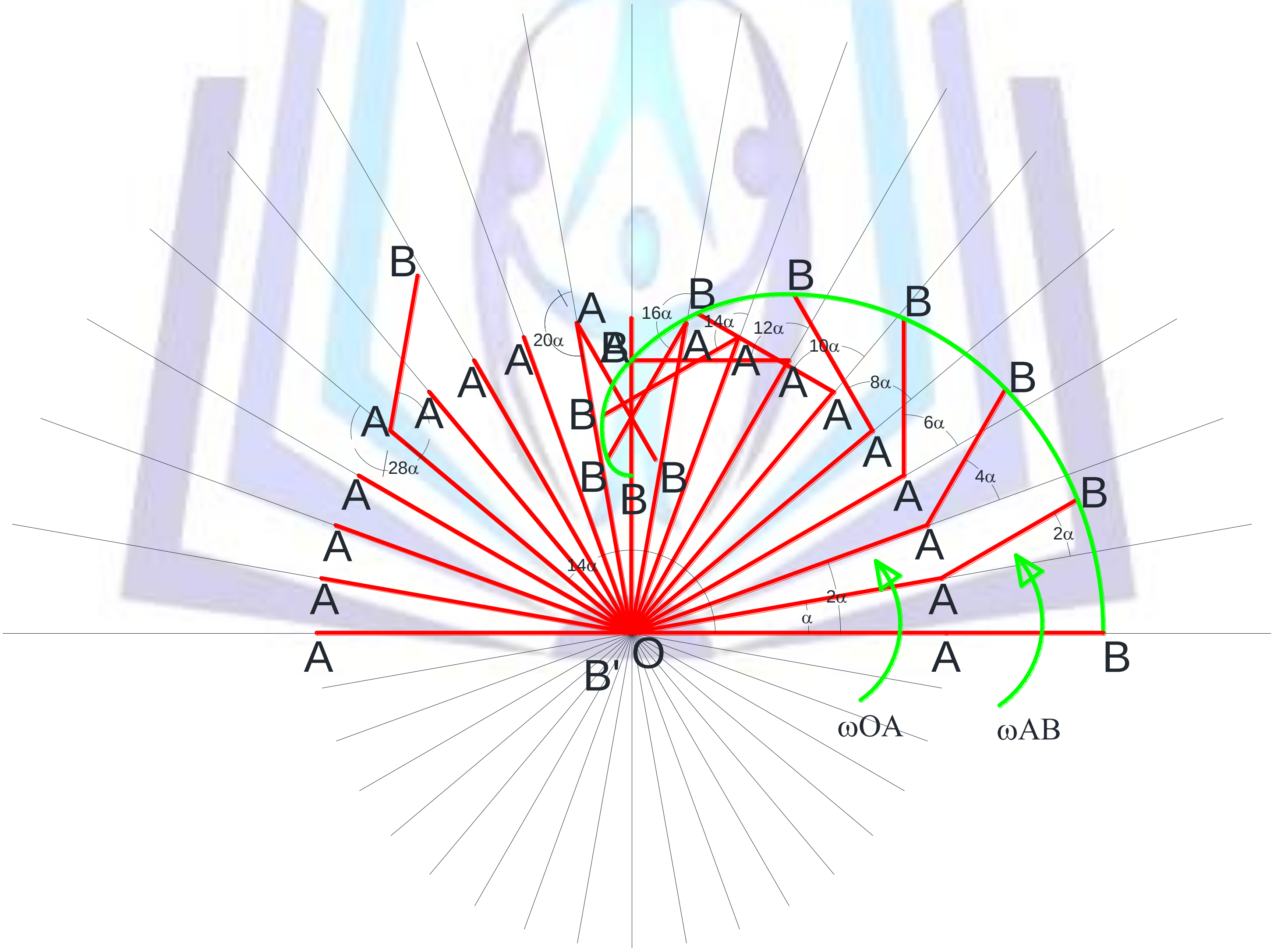
Ex 6

Codirectional angular speed

$$\begin{cases} \omega AB = 2.\omega OA \Rightarrow \omega AB = \dfrac{\pi}{18} Rad/Sec \\ t = 1\,sec \\ \overline{OA} = \overline{2.AB} \\ \omega OB = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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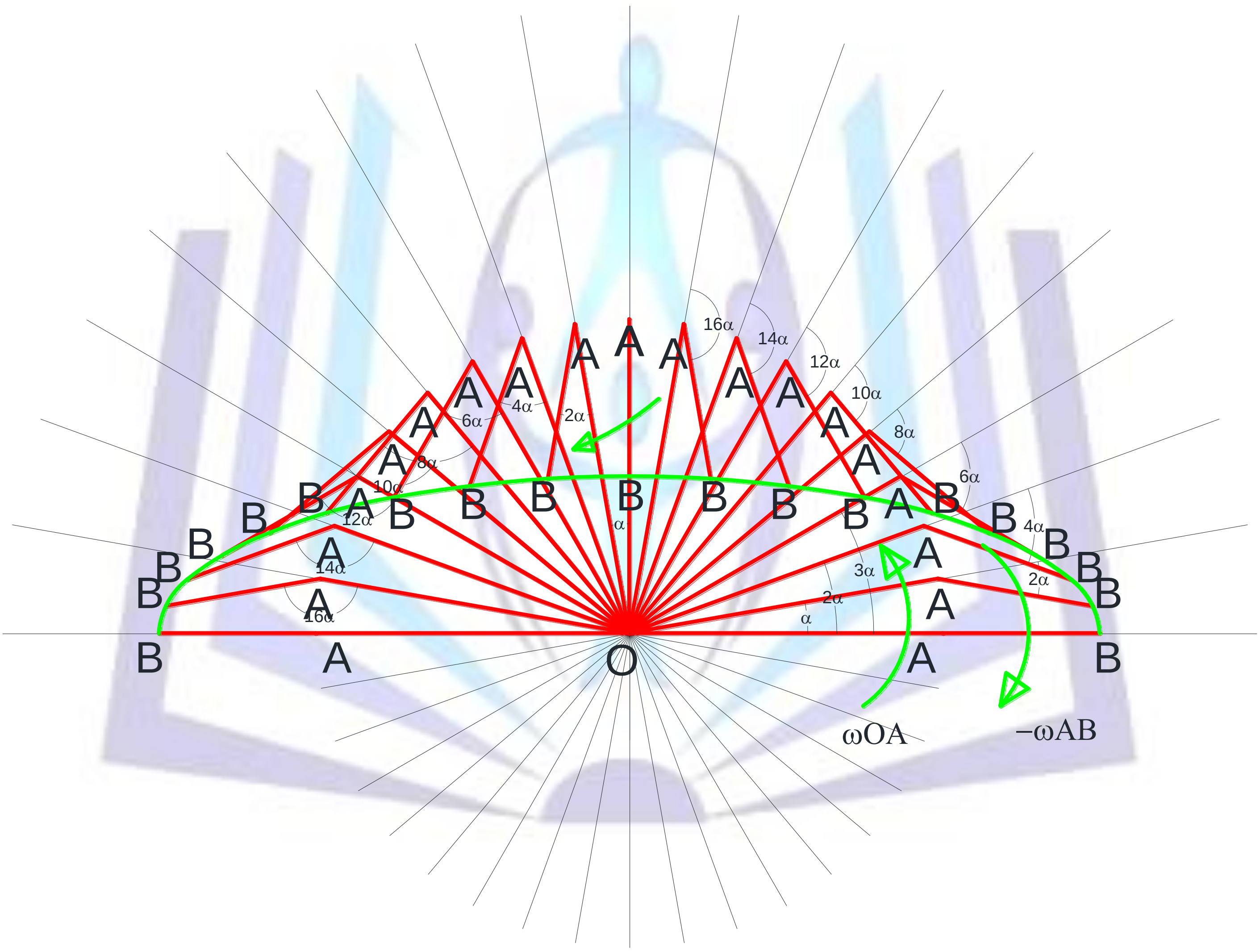
Ex 7

$$\begin{cases} \omega AB = 2.\omega OA \Rightarrow \omega OA = \dfrac{\pi}{18} Rad/Sec \\ t = 1\,sec \\ \overline{OA} = \overline{2.AB} \\ \omega OB = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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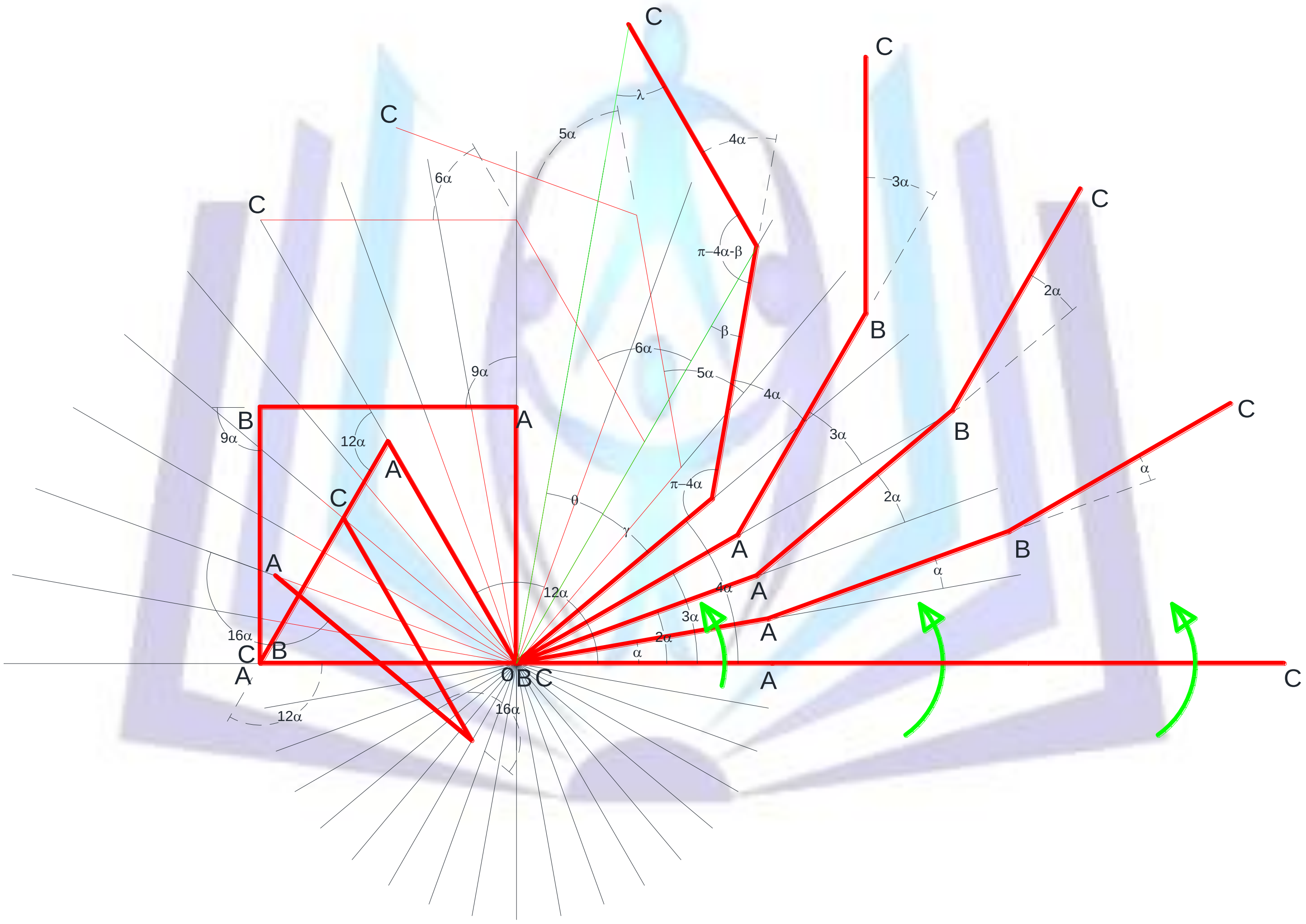
Ex 8

$$\begin{cases} \omega BC = \omega AB = \omega OA \Rightarrow \omega OA \dfrac{\pi}{18} Rad/Sec \\ t = 1\,sec \\ \overline{OA} = \overline{AB} = \overline{BC} \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$

$$WOC = (4\alpha + \gamma + \theta)$$

T=4 Sec



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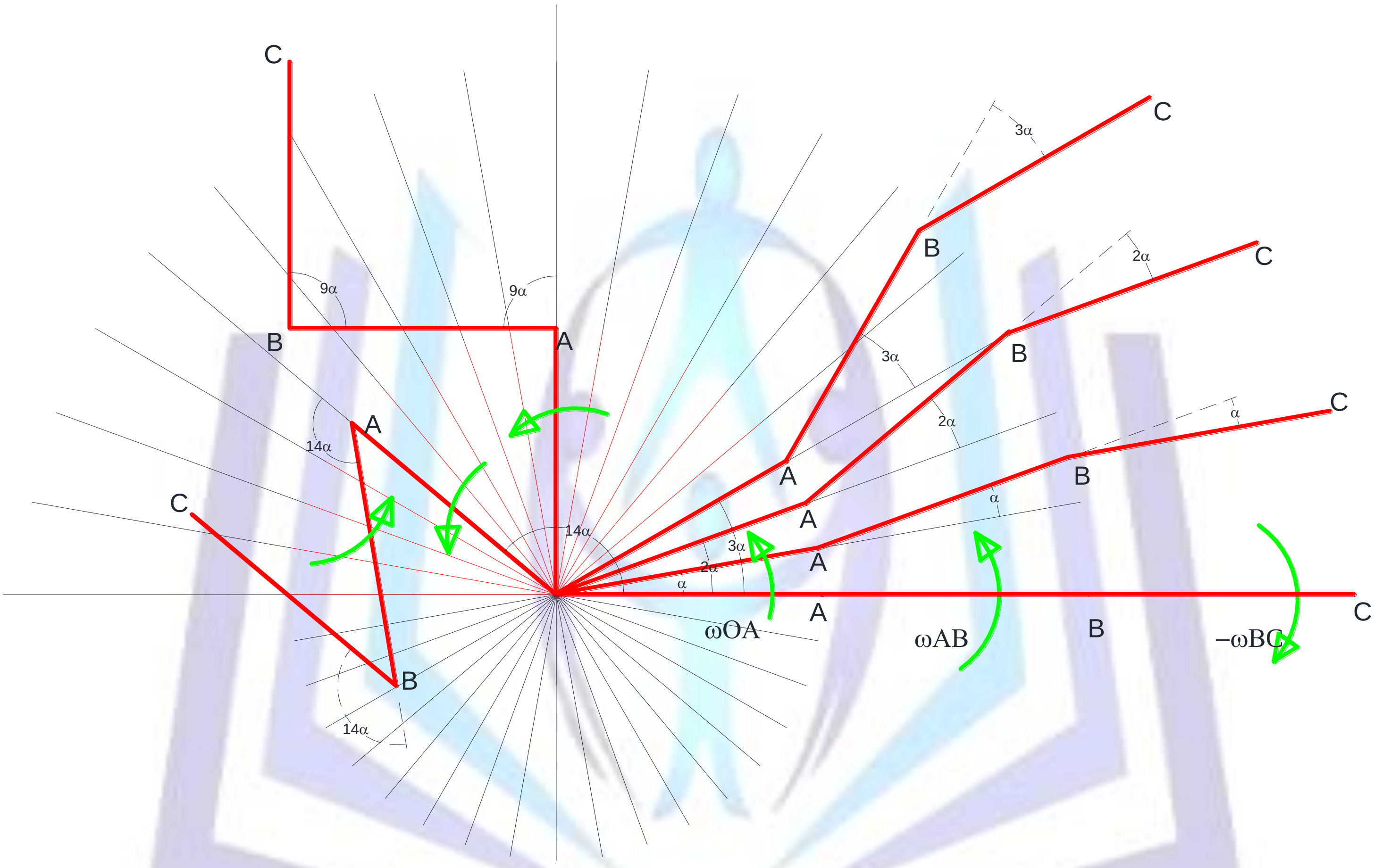
Ex 9

$$\begin{cases} -\omega BC = \omega AB = \omega OA \Rightarrow \omega OA \dfrac{\pi}{18} Rad/Sec \\ t = 1\,sec \\ \overline{OA} = \overline{AB} = \overline{BC} \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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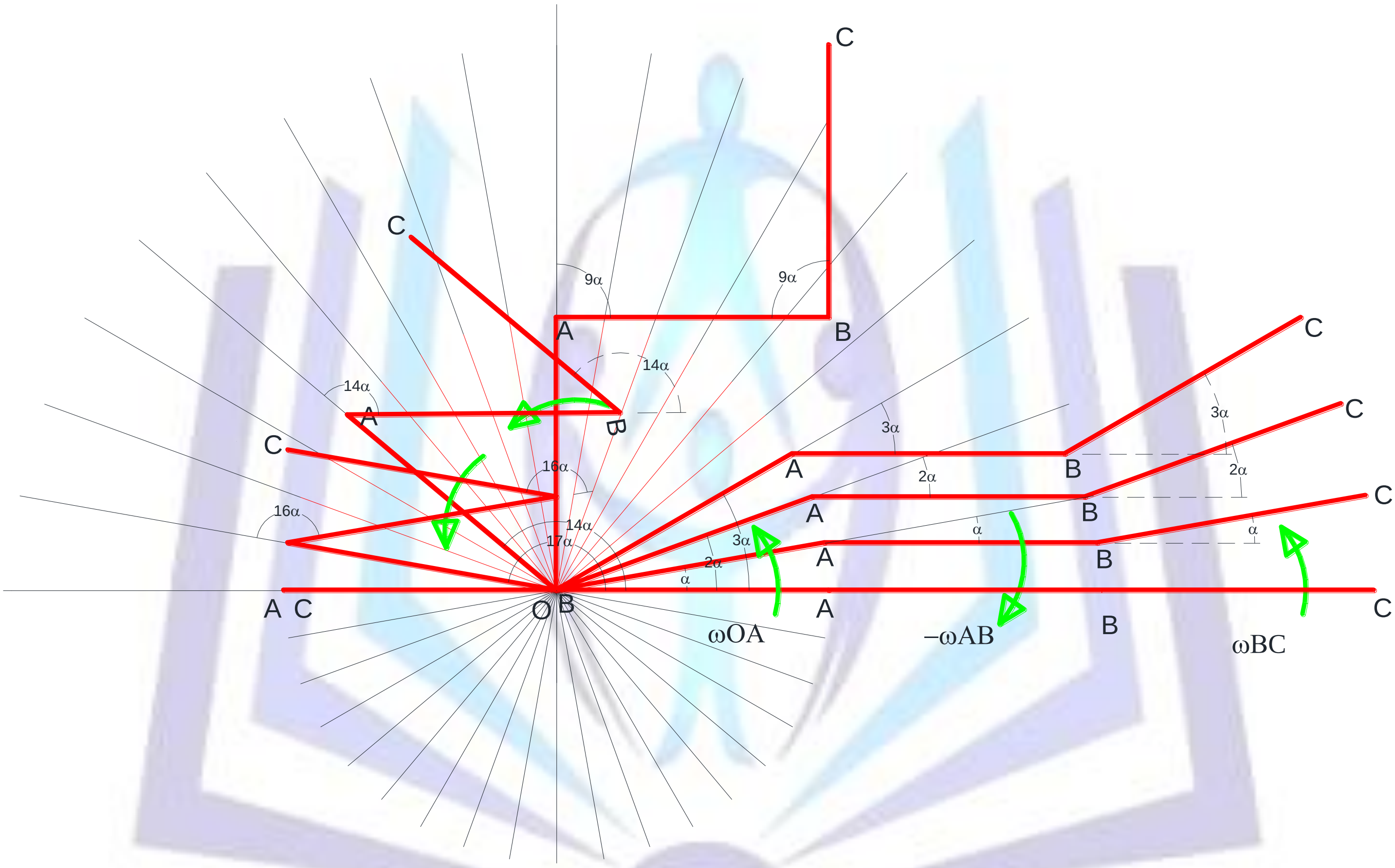
Ex 10

$$\begin{cases} \omega BC = -\omega AB = \omega OA \Rightarrow \omega OA \dfrac{\pi}{18} Rad/Sec \\ t = 1\,sec \\ \overline{OA} = \overline{AB} = \overline{BC} \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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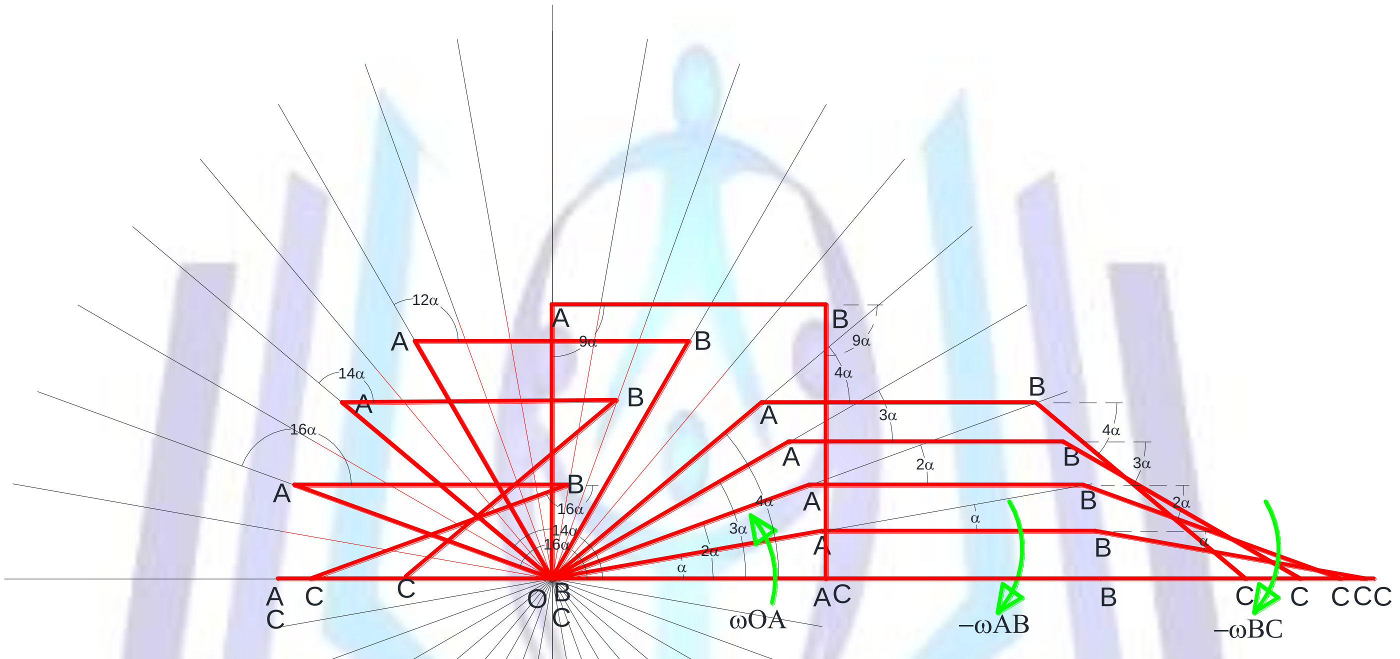
Ex 11

$$\begin{cases} -\omega BC = -\omega AB = \omega OA \Rightarrow \omega OA \dfrac{\pi}{18} Rad/Sec \\ t = 1\,sec \\ \overline{OA} = \overline{AB} = \overline{BC} \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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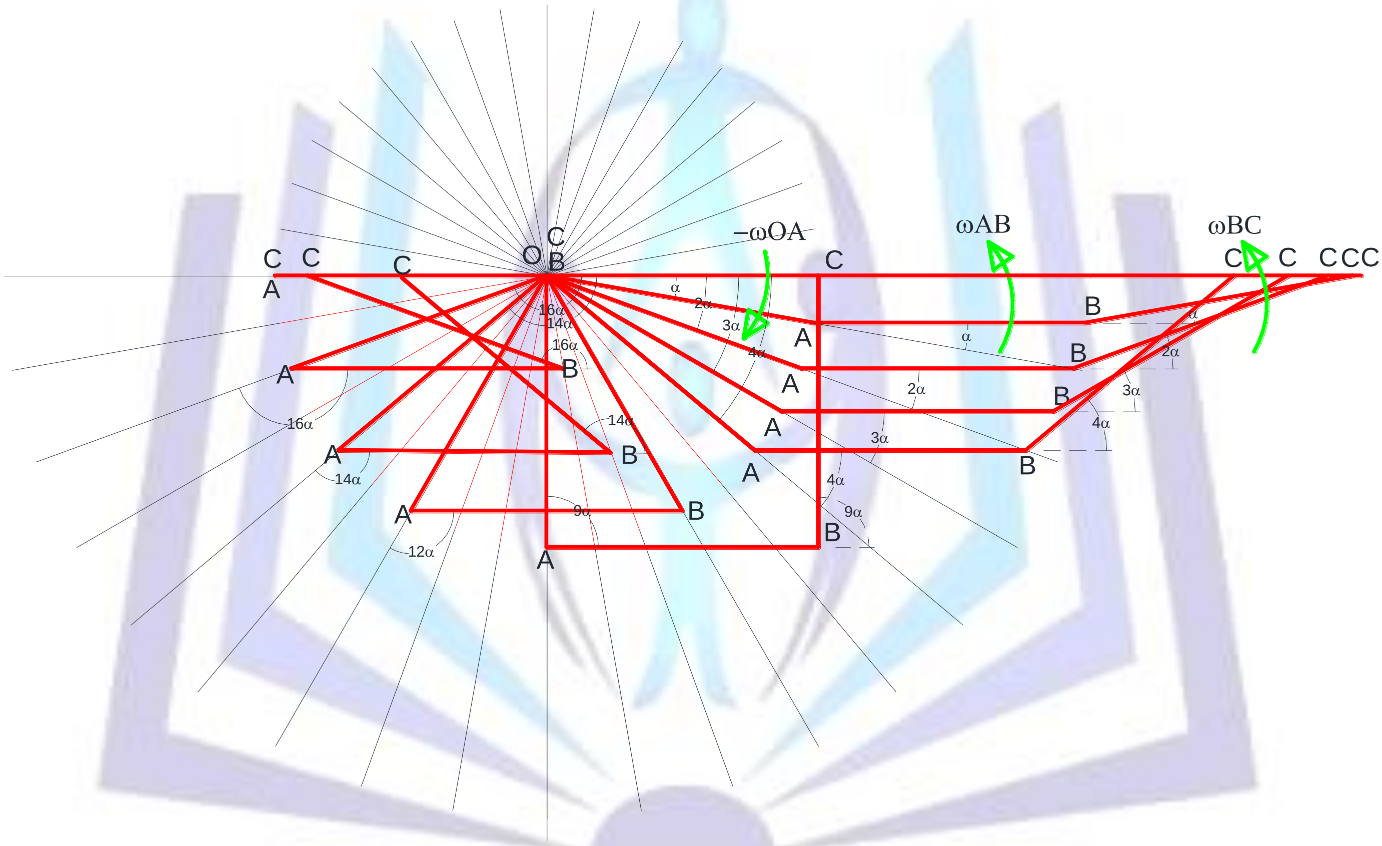
Ex 12

$$\begin{cases} \omega BC = \omega AB = -\omega OA \Rightarrow \omega OA = \dfrac{-\pi}{18} Rad/Sec \\ t = 1\,sec \\ \overline{OA} = \overline{AB} = \overline{BC} \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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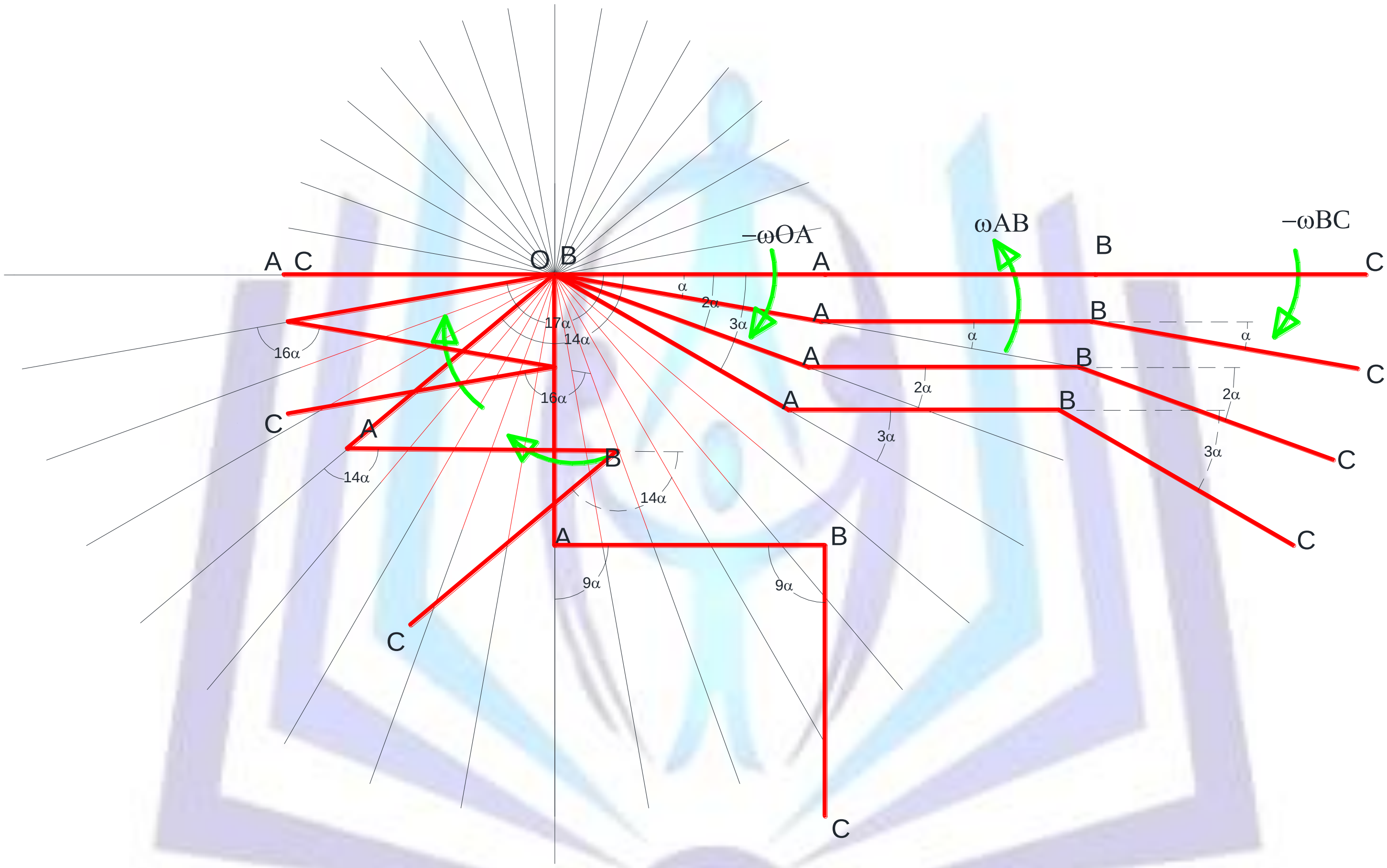
Ex 13

$$\begin{cases} -\omega BC = \omega AB = -\omega OA \Rightarrow \omega OA = \dfrac{-\pi}{18} Rad/Sec \\ t = 1\ sec \\ \overline{OA} = \overline{AB} = \overline{BC} \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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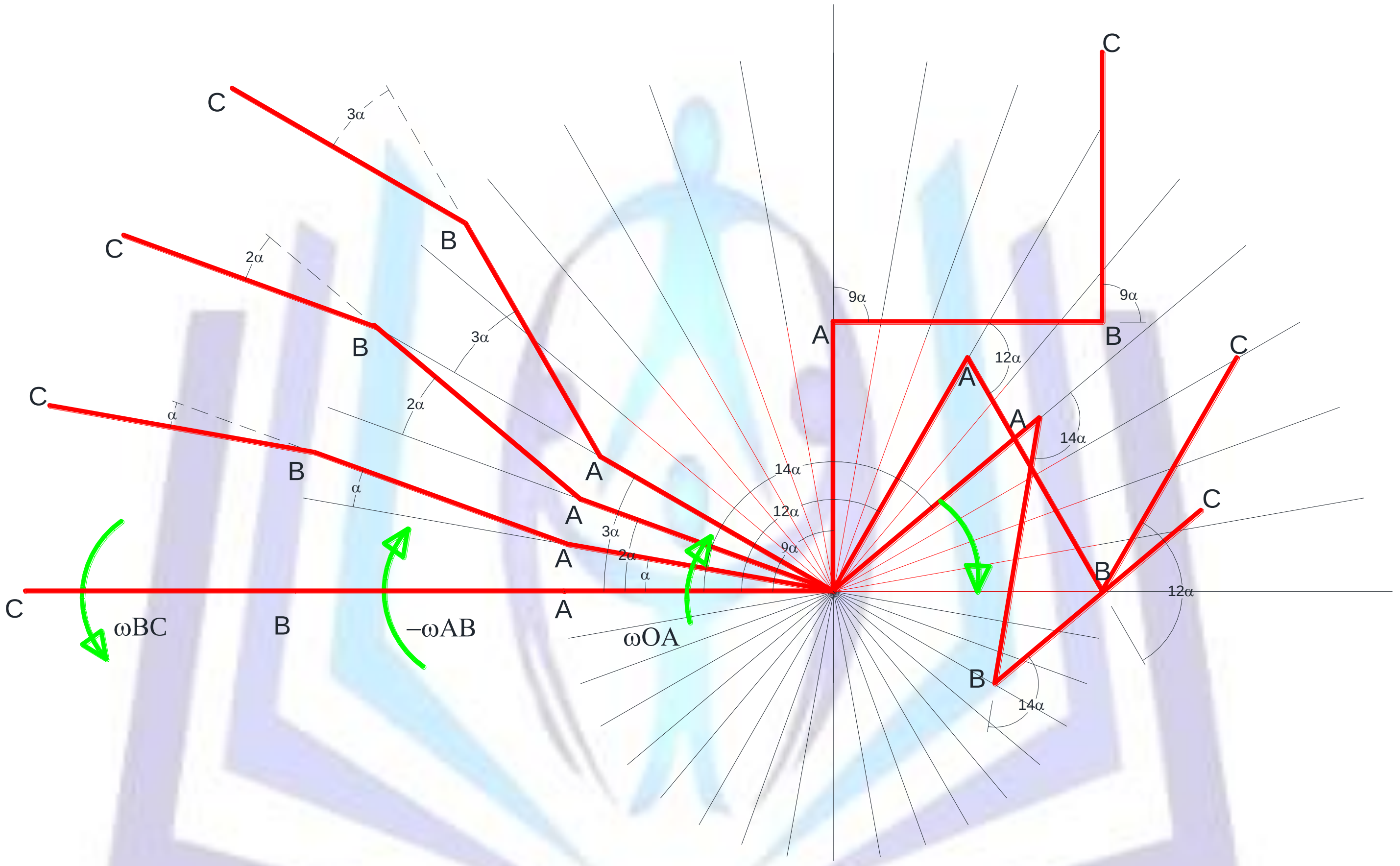
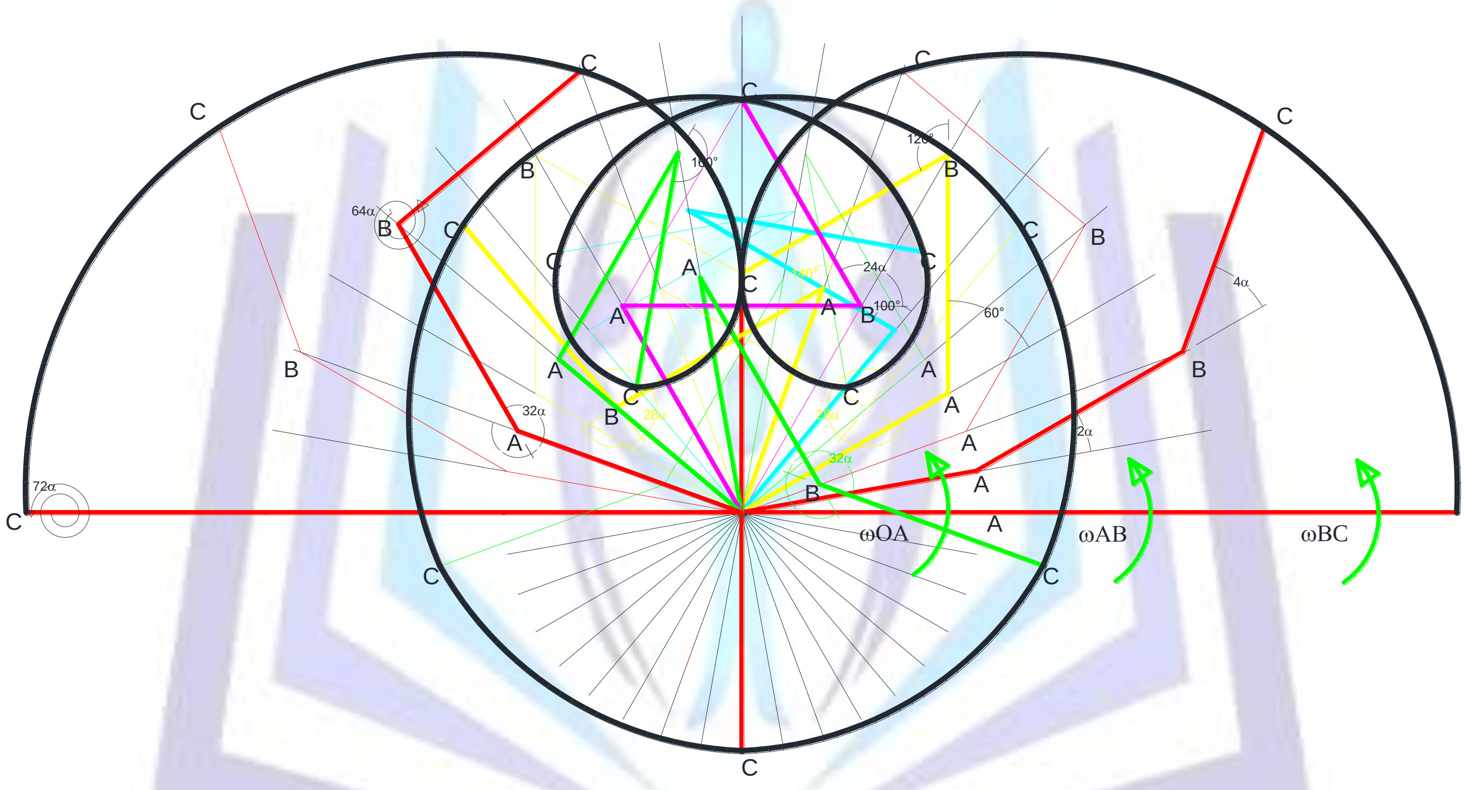
Ex 14

$$\begin{cases} \omega BC = -\omega AB = -\omega OA \Rightarrow \omega OA = \dfrac{\pi}{18} Rad/Sec \\ \quad t = 1\,sec \\ \qquad \overline{OA} = \overline{AB} = \overline{BC} \\ \qquad\qquad \omega OC = ? \\ \qquad\qquad\quad \alpha = \dfrac{\pi}{18} \end{cases}$$



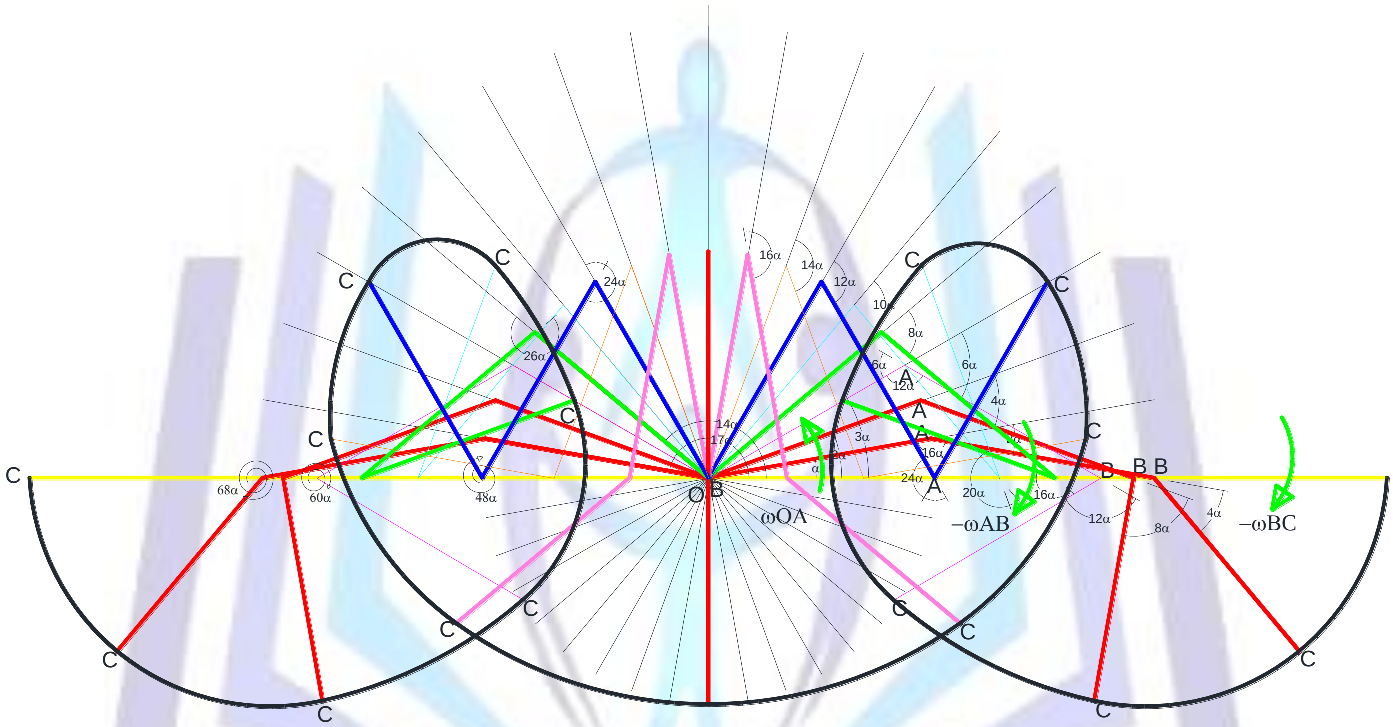
Ex 15

$$\begin{cases} \boldsymbol{\omega} OA = \frac{\pi}{18}\ Rad/Sec \\ t = 1\,sec \\ \overline{OA} = \overline{AB} = \overline{BC} \\ \boldsymbol{\omega} BC = 2.\omega AB \Rightarrow \omega AB = 2.\omega OA \\ \alpha = \frac{\pi}{18} \end{cases}$$



Ex 16

$$\begin{cases} \boldsymbol{\omega} OA = \dfrac{\pi}{18}\ Rad/Sec \\ t = 1\,sec \\ \overline{OA} = \overline{AB} = \overline{BC} \\ -\omega BC = 2.\omega AB \ \Rightarrow \ 2.\omega AB = -\omega OA \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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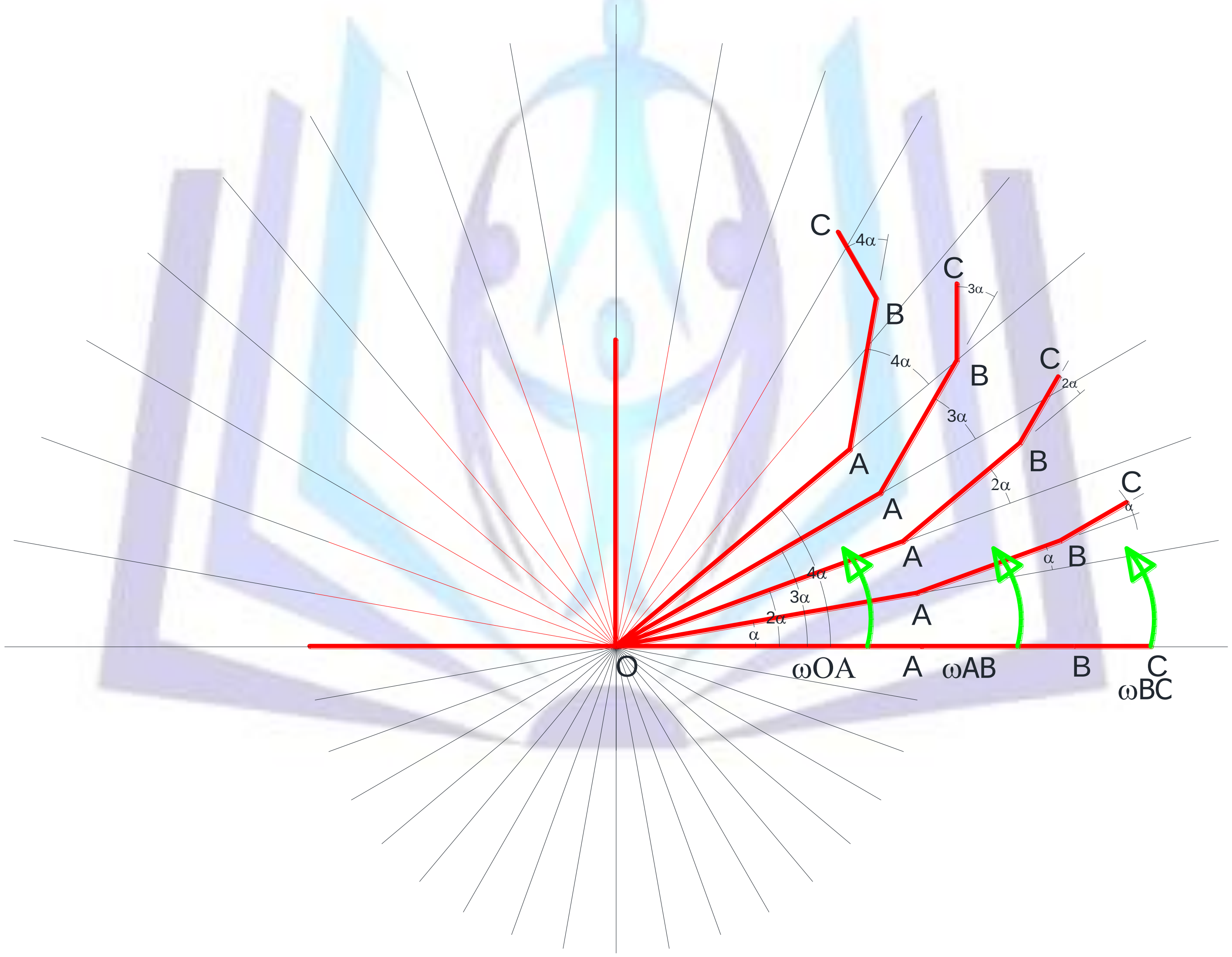
Ex 17

$$\begin{cases} \boldsymbol{\omega} OA = \frac{\pi}{18}\, Rad/Sec \\ t = 1\, sec \\ \overline{OA} = \overline{2.AB} = \overline{4.BC} \\ \omega BC = \omega AB = \omega OA \\ \omega OC = ? \\ \alpha = \frac{\pi}{18} \end{cases}$$



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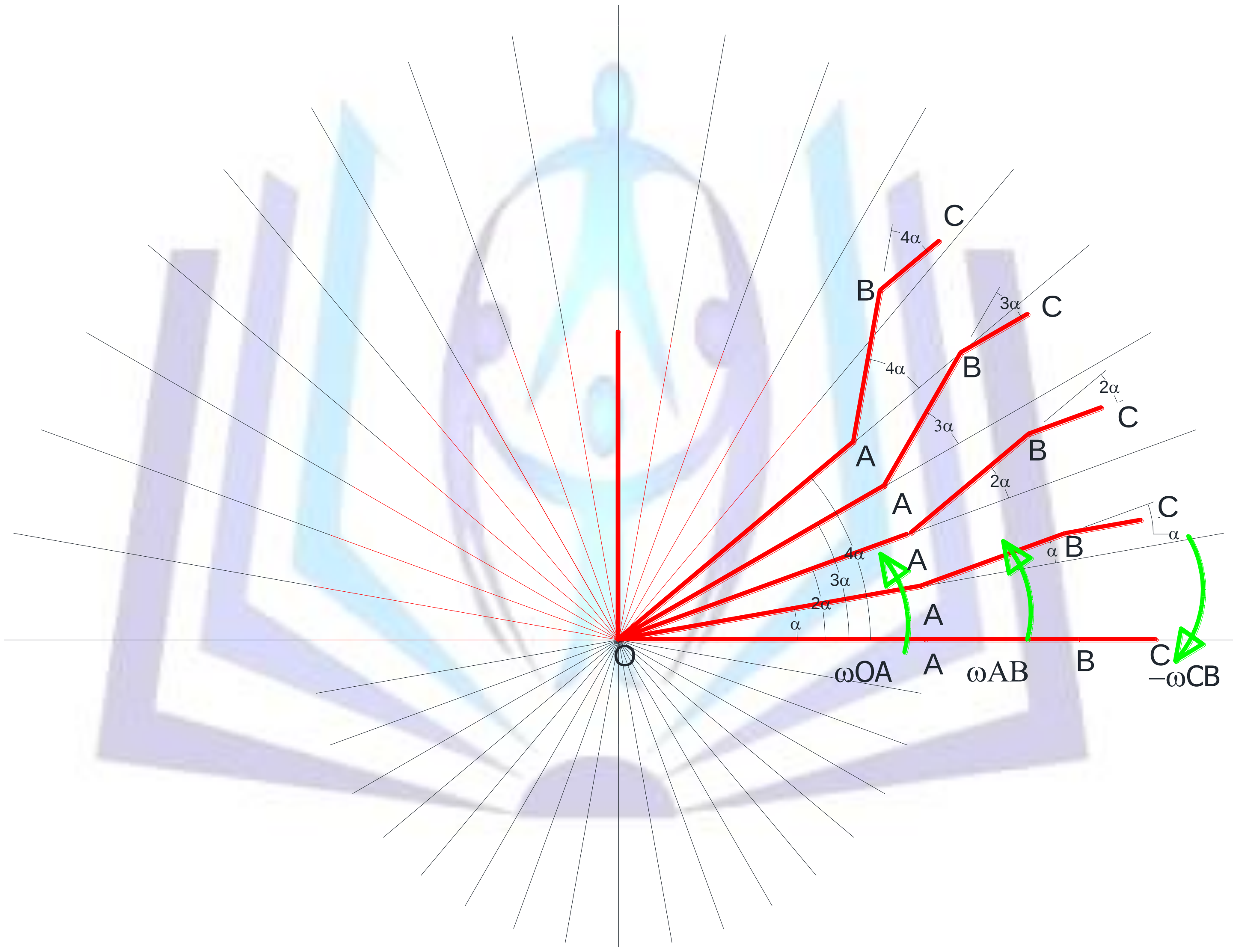
Ex 18

$$\begin{cases} \boldsymbol{\omega} OA = \dfrac{\pi}{18}\ Rad/Sec \\ t = 1\,sec \\ \overline{OA} = \overline{2.AB} = \overline{4.BC} \\ \omega BC = \omega AB = \omega BC \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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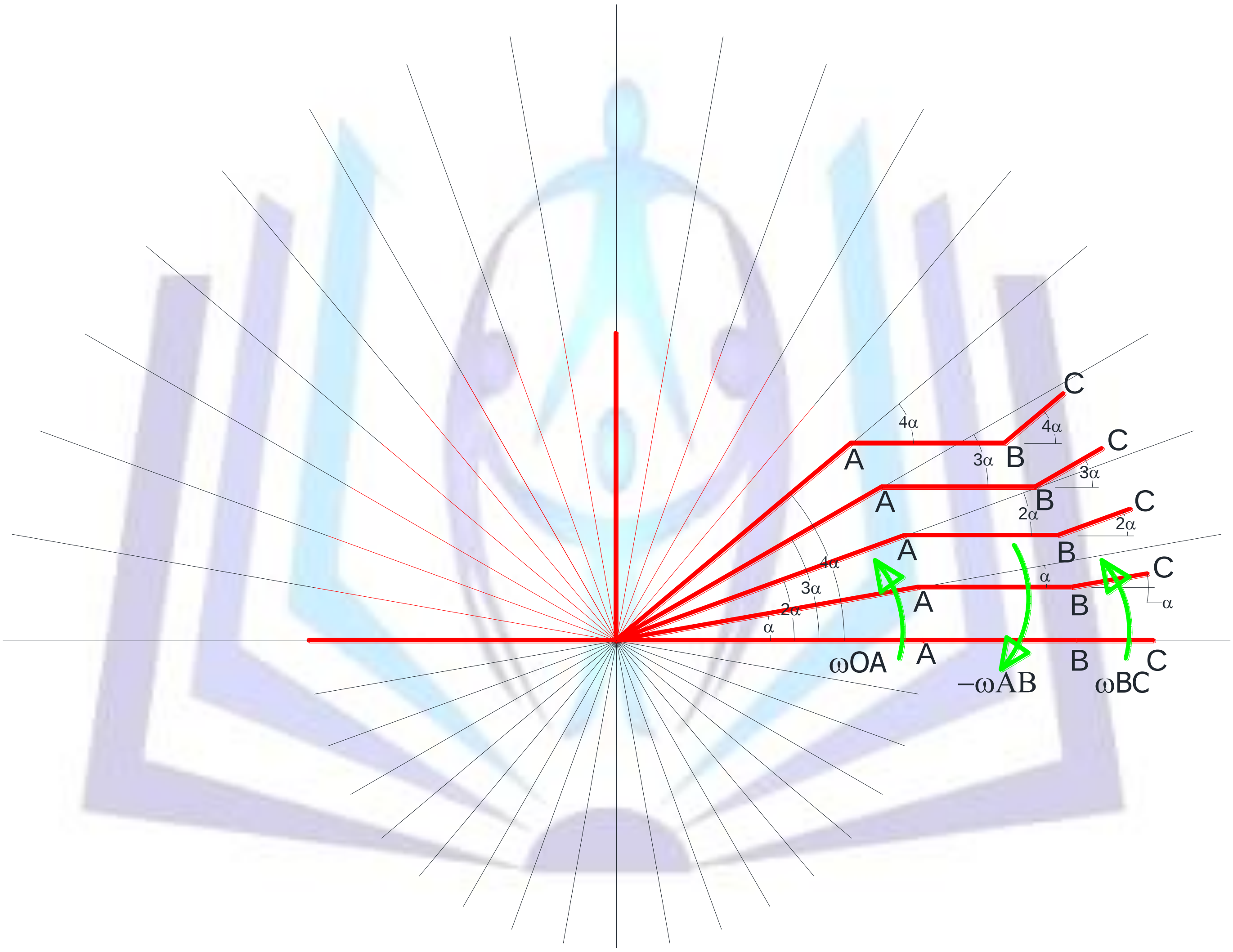
Ex 19

$$\begin{cases} \boldsymbol{\omega} OA = \dfrac{\pi}{18}\ Rad/Sec \\ t = 1\,sec \\ \overline{OA} = \overline{2.AB} = \overline{4.BC} \\ \omega OA = -\omega AB = \omega BC \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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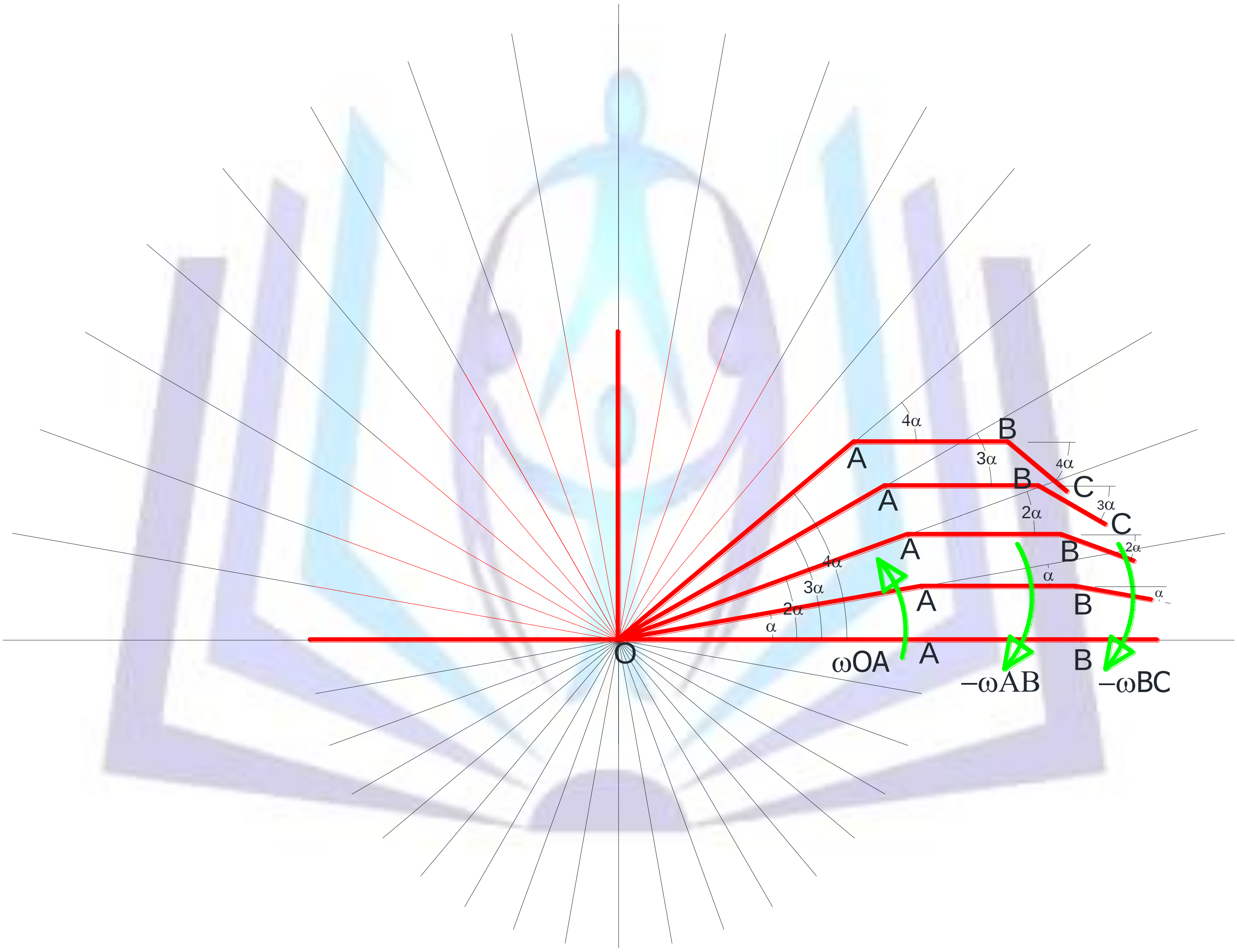
Ex 20

$$\begin{cases} \boldsymbol{\omega} OA = \dfrac{\pi}{18}\ Rad/Sec \\ t = 1\,sec \\ \overline{OA} = \overline{2.AB} = \overline{4.BC} \\ \omega OA = -\omega AB = -\omega BC \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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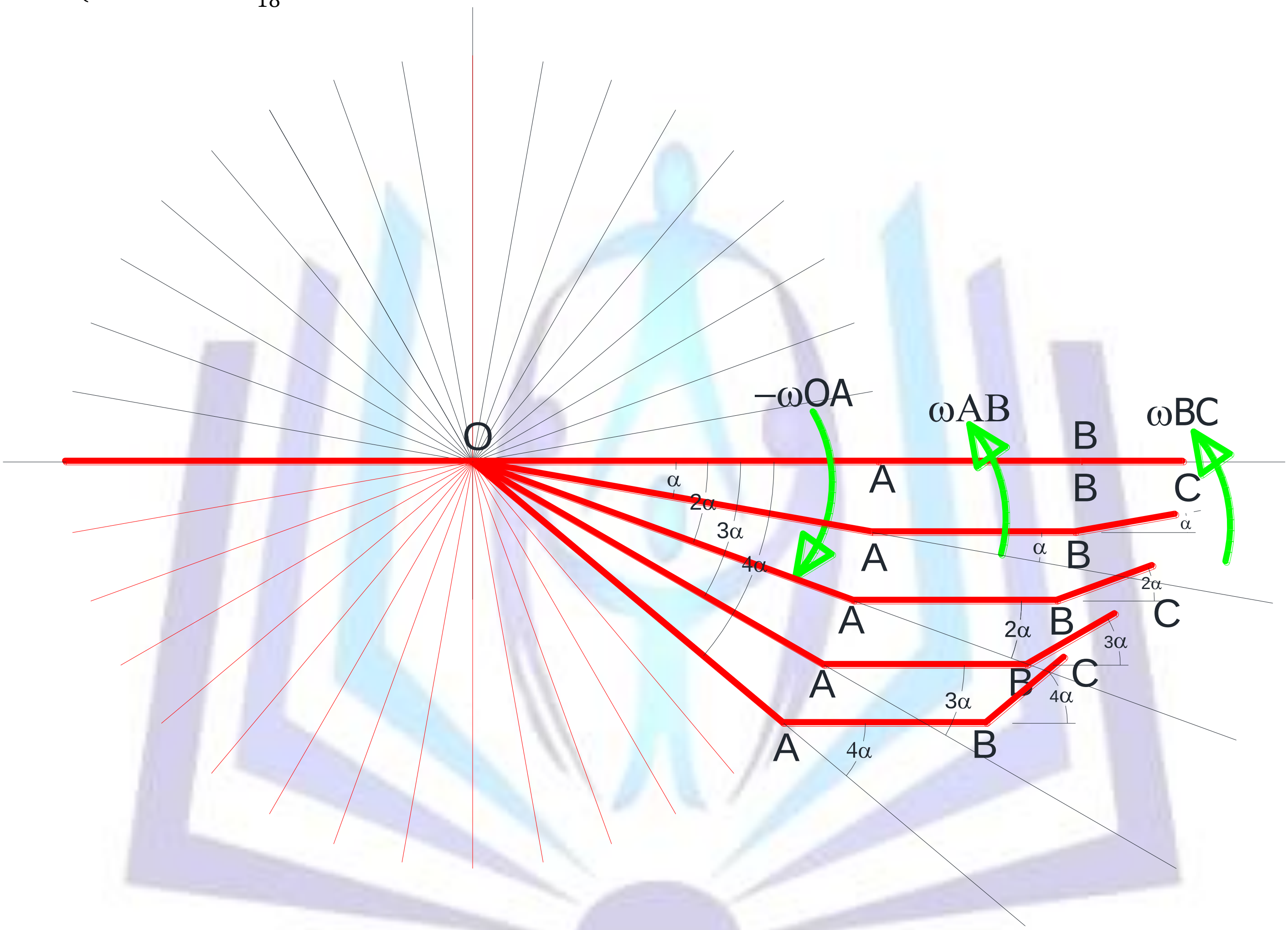
Ex 21

$$\begin{cases} \omega OA = \frac{-\pi}{18}\ Rad\ /Sec \\ t = 1\,sec \\ \overline{OA} = \overline{2.AB} = \overline{4.BC} \\ -\omega OA = \omega AB = \omega BC \\ \omega OC = ? \\ \alpha = \frac{\pi}{18} \end{cases}$$



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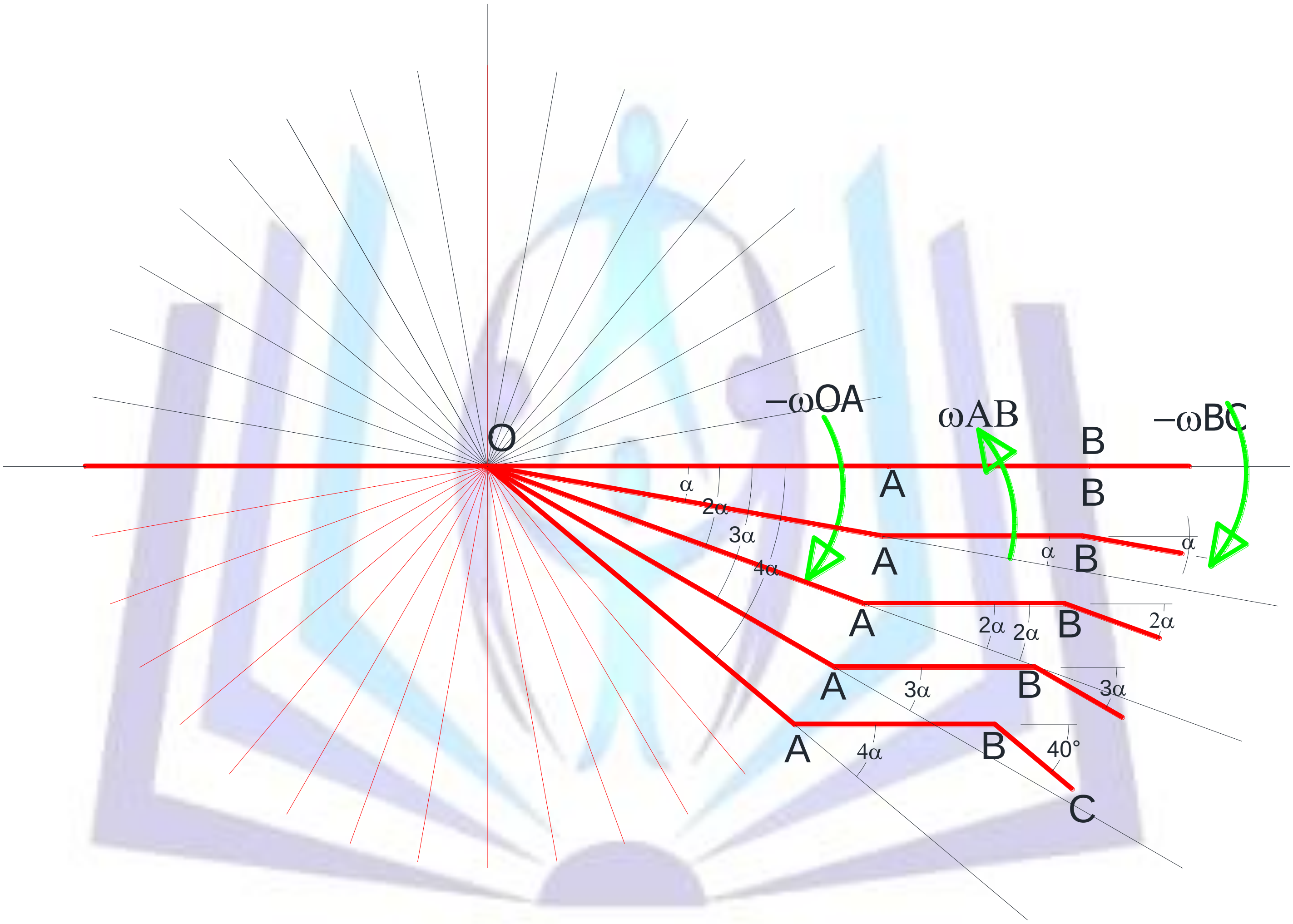
Ex 22

$$\begin{cases} \boldsymbol{\omega} OA = \dfrac{\pi}{18}\ Rad/Sec \\ t = 1\,sec \\ \overline{OA} = \overline{2.AB} = \overline{4.BC} \\ -\omega OA = \omega AB = -\omega BC \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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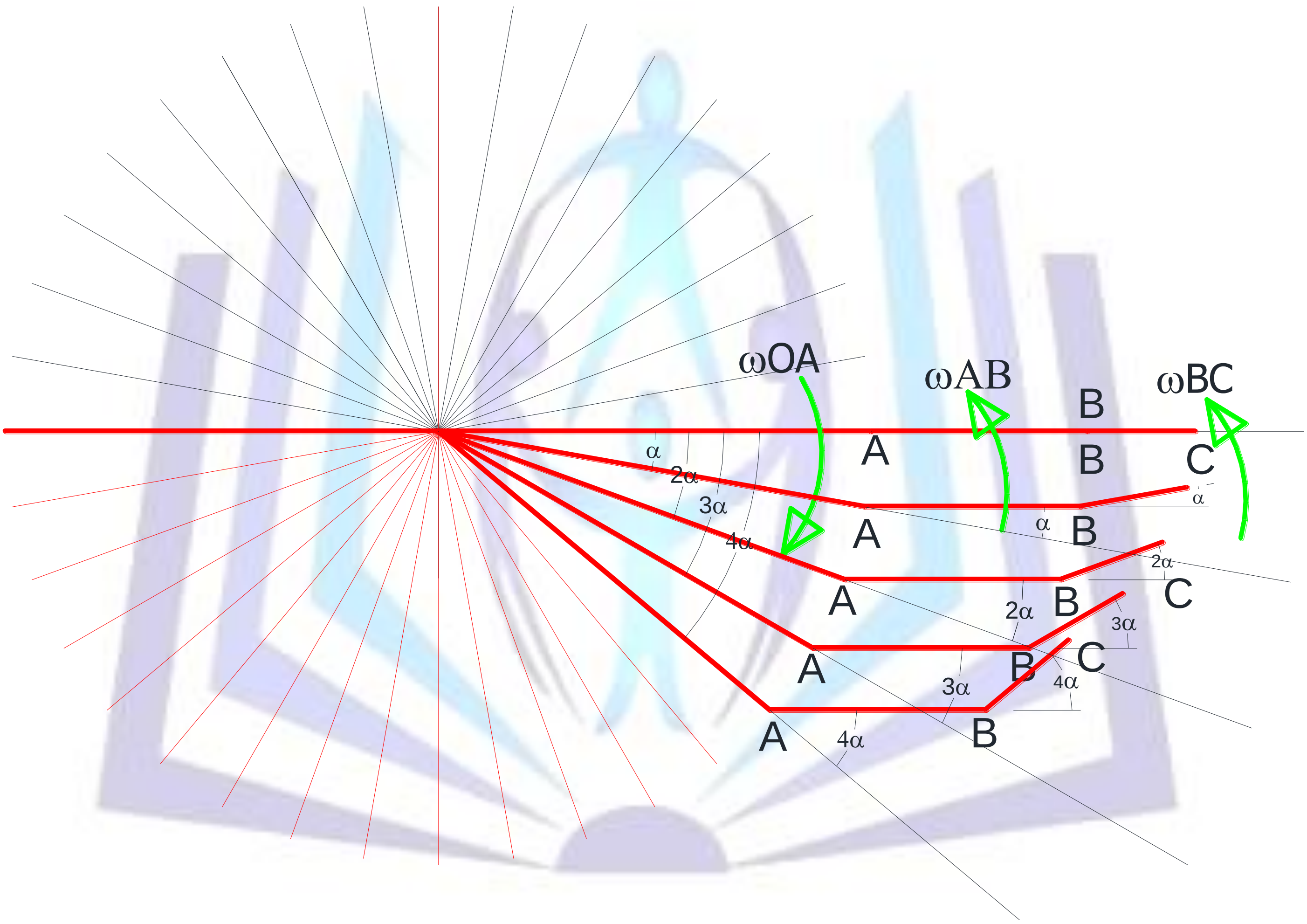
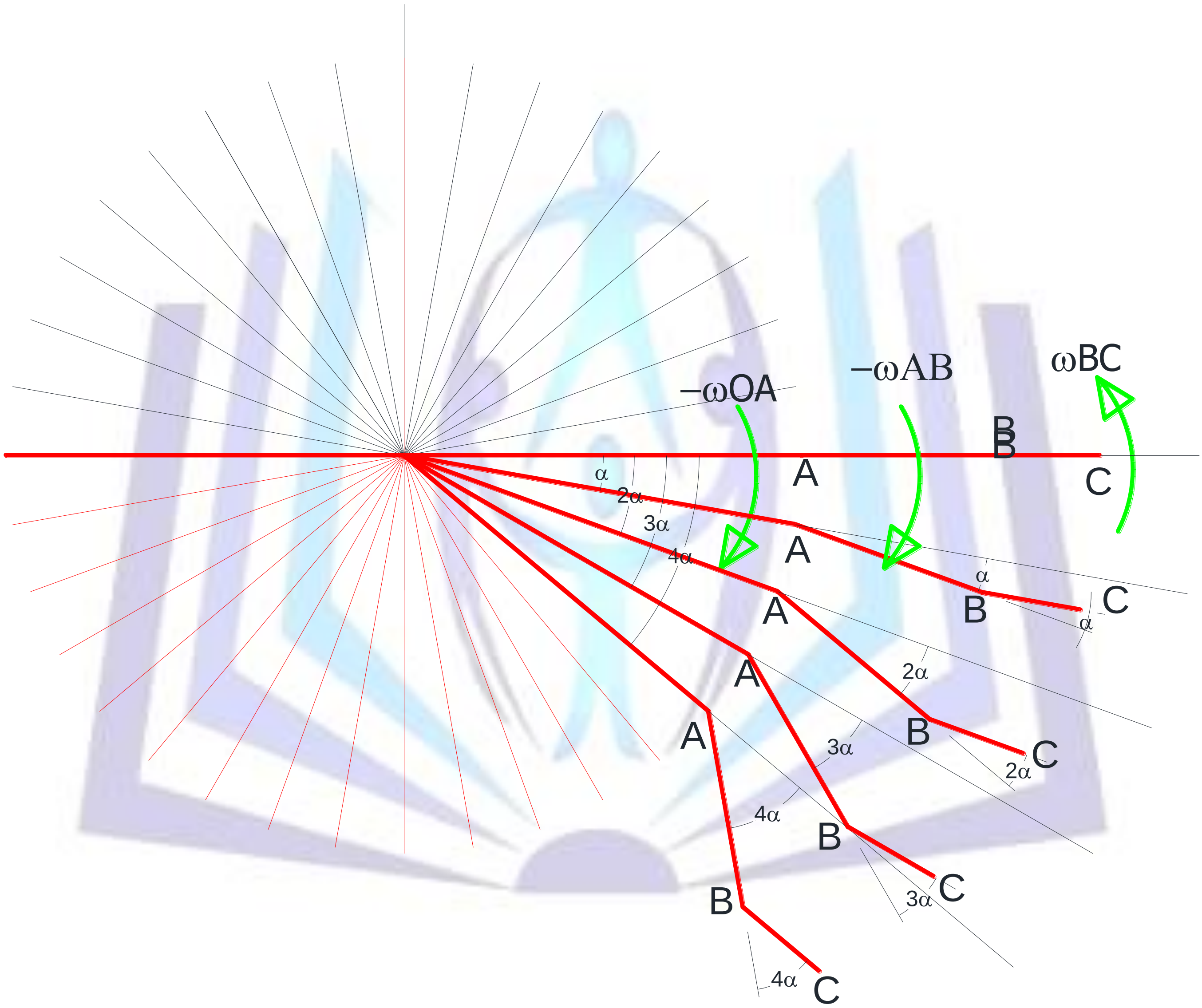
Ex 23

$$\begin{cases} \boldsymbol{\omega OA} = \frac{\pi}{18}\ Rad/Sec \\ t = 1\,sec \\ \overline{OA} = \overline{2.AB} = \overline{4.BC} \\ -\omega OA = \omega AB = \omega BC \\ \omega OC = ? \\ \alpha = \frac{\pi}{18} \end{cases}$$



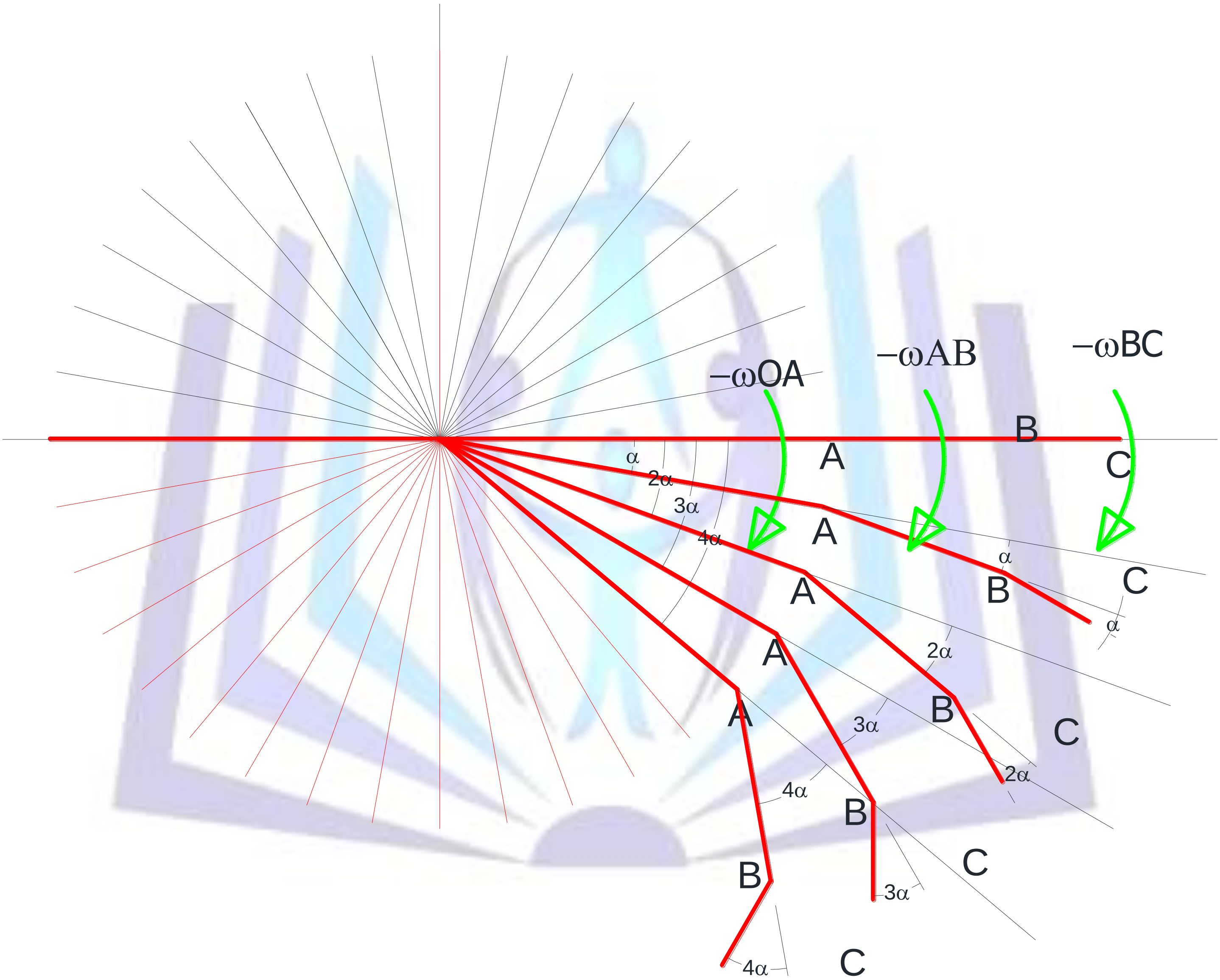
Ex 24

$$\begin{cases} \omega OA = \frac{-\pi}{18}\ Rad/Sec \\ t = 1\,sec \\ \overline{OA} = \overline{2.AB} = \overline{4.BC} \\ -\omega OA = -\omega AB = \omega BC \\ \omega OC = ? \\ \alpha = \frac{\pi}{18} \end{cases}$$



Ex 25

$$\begin{cases} \omega OA = \dfrac{-\pi}{18}\ Rad/Sec \\ t = 1\,sec \\ \overline{OA} = \overline{2.AB} = \overline{4.BC} \\ -\omega OA = -\omega AB = -\omega BC \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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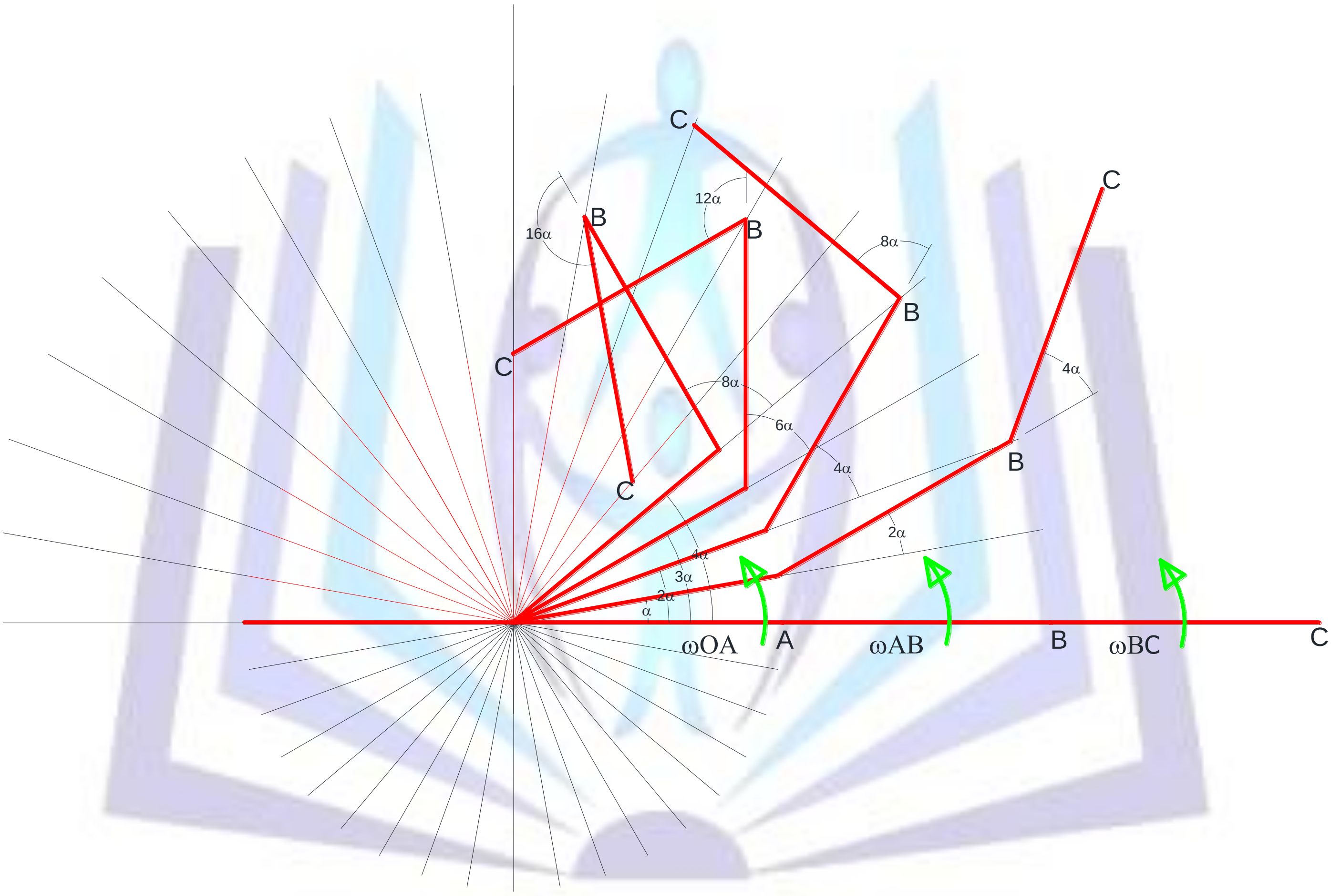
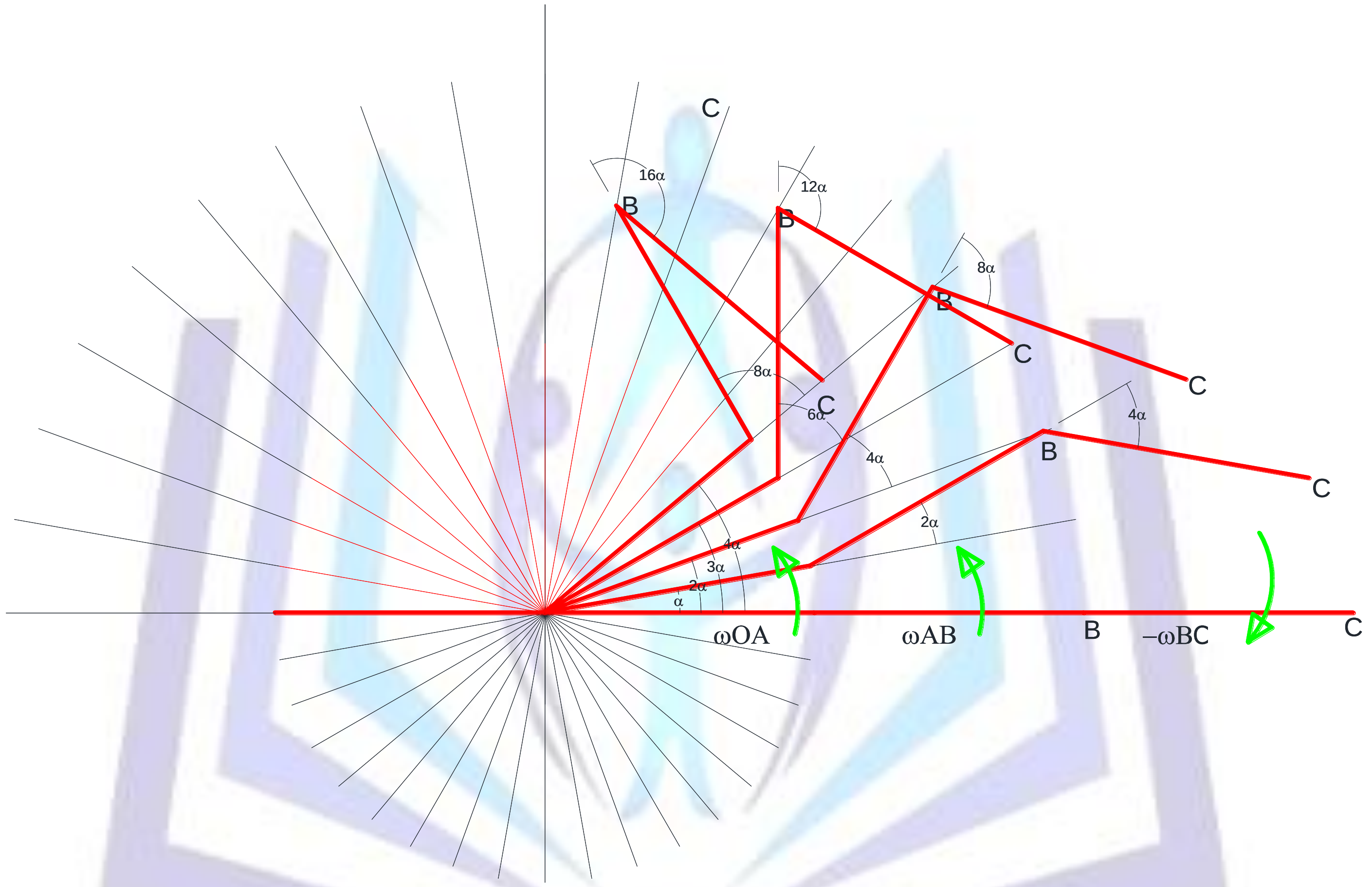
Ex 26

$$\begin{cases} \boldsymbol{\omega} OA = \dfrac{\pi}{18}\ Rad/Sec \\ t = 1\,sec \\ \overline{OA} = \overline{AB} = \overline{BC} \\ \omega OA = 2.\omega AB = 4.\omega BC \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



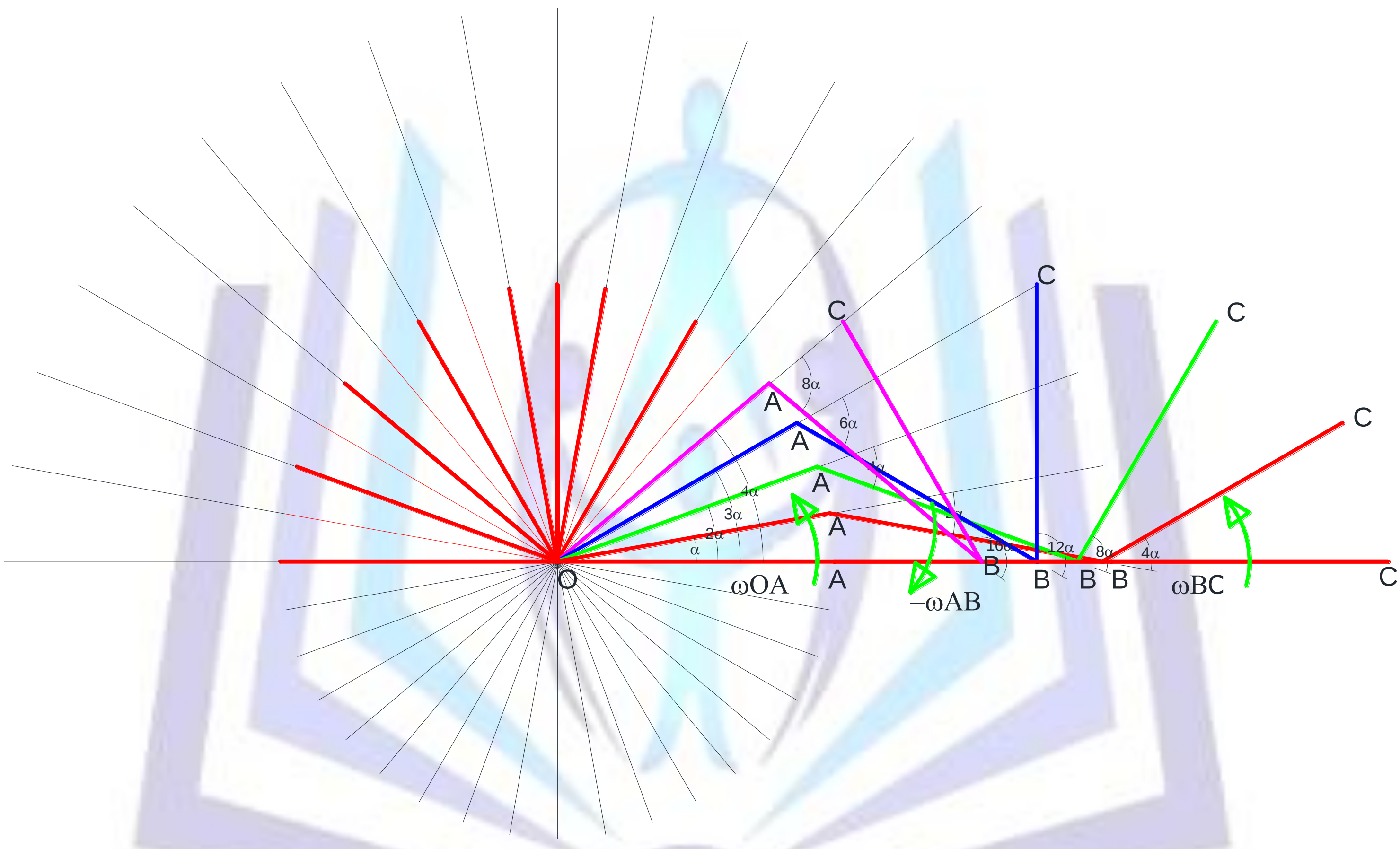
<u>**Ex 27**</u>

$$\begin{cases} \boldsymbol{\omega} OA = \dfrac{\pi}{18}\ Rad/Sec \\ t = 1\ sec \\ \overline{OA} = \overline{AB} = \overline{BC} \\ \omega OA = 2.\omega AB = 4.\omega BC \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



Ex 28

$$\begin{cases} \boldsymbol{\omega} OA = \dfrac{\pi}{18}\ Rad/Sec \\ t = 1\ sec \\ \overline{OA} = \overline{AB} = \overline{BC} \\ \omega OA = -2.\omega AB = 4.\omega BC \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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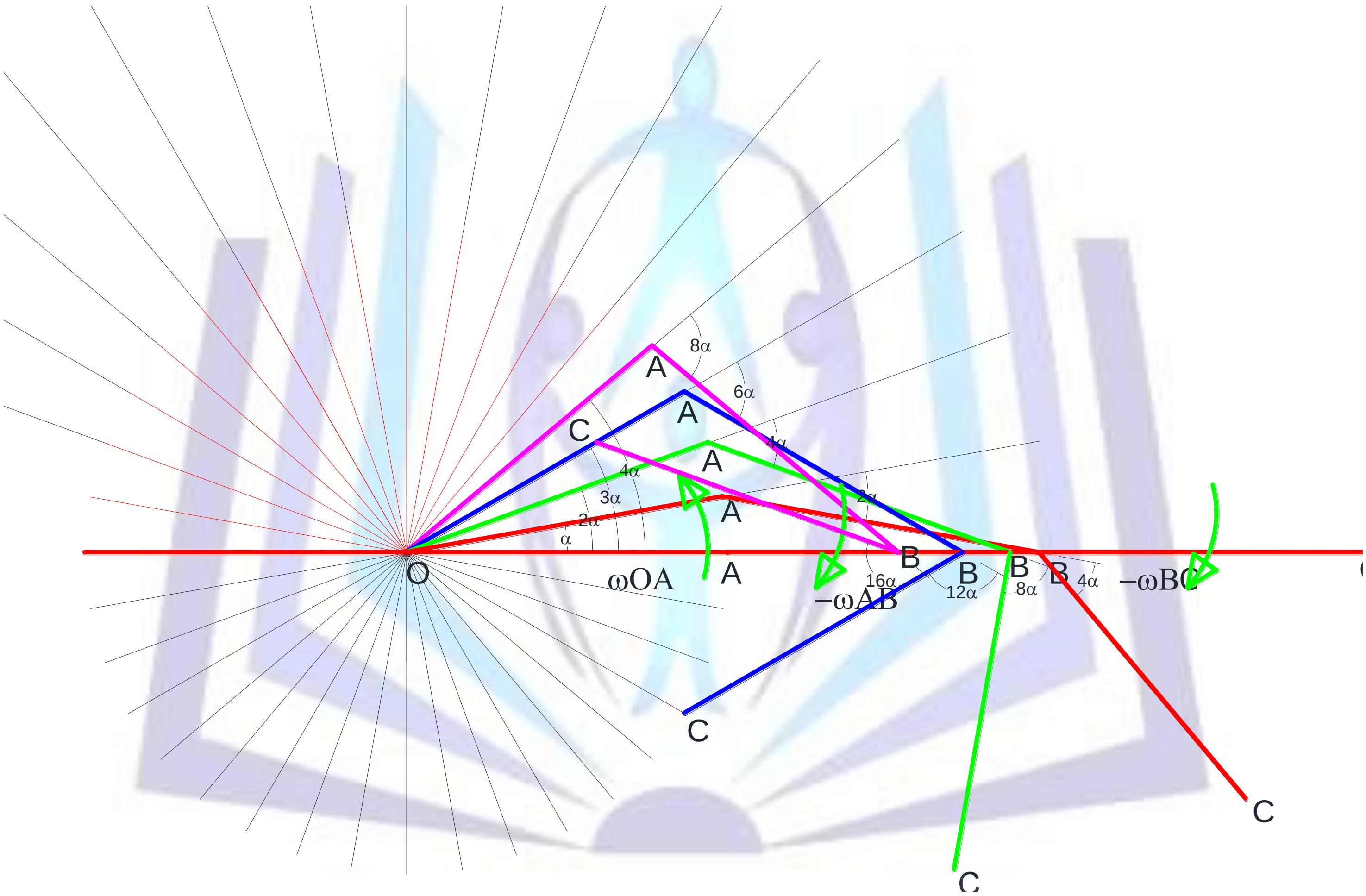
Ex 29

$$\begin{cases} \boldsymbol{\omega} OA = \dfrac{\pi}{18}\ Rad/Sec \\ t = 1\ sec \\ \overline{OA} = \overline{AB} = \overline{BC} \\ \omega OA = -2.\omega AB = -4.\omega BC \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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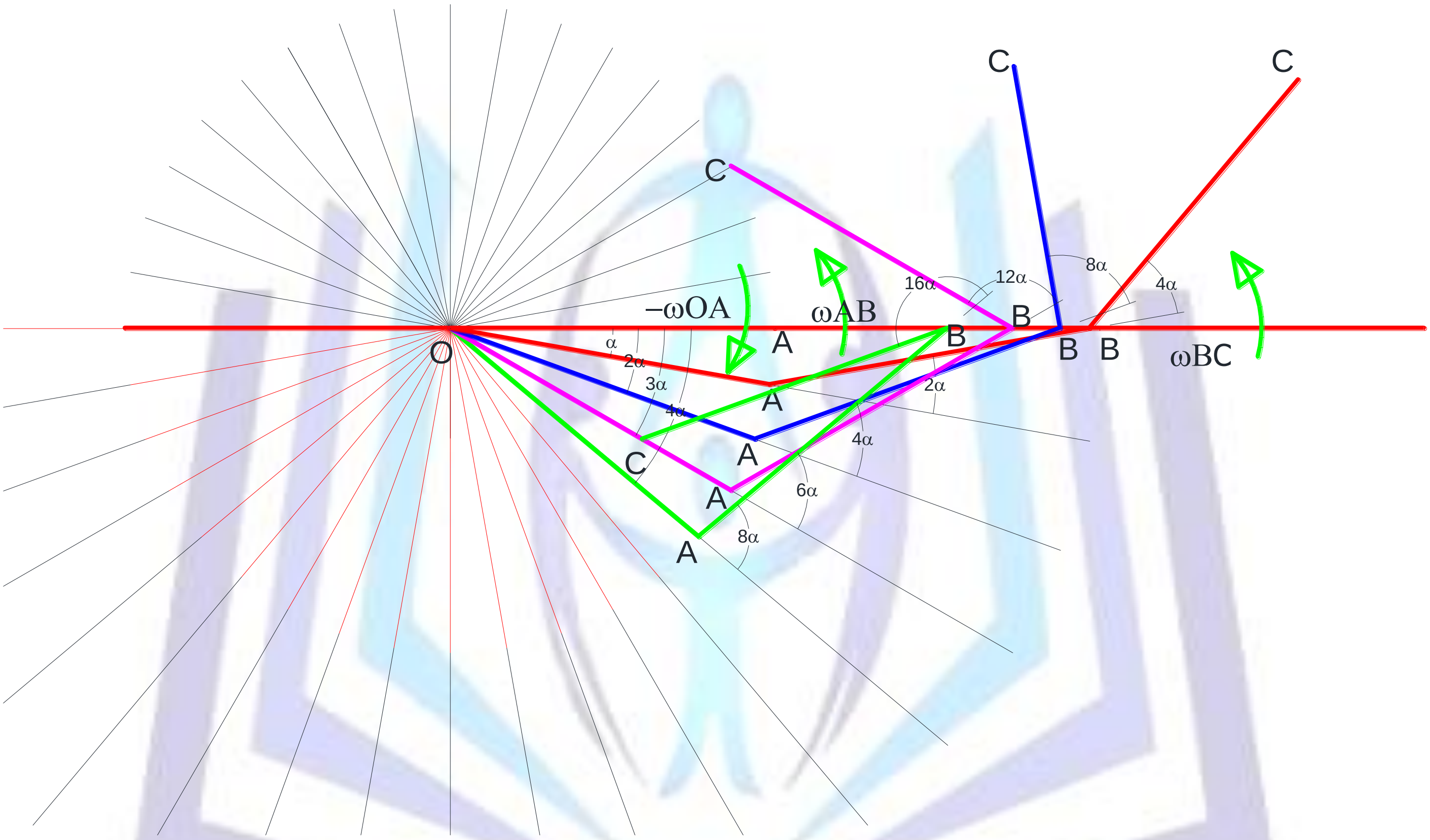
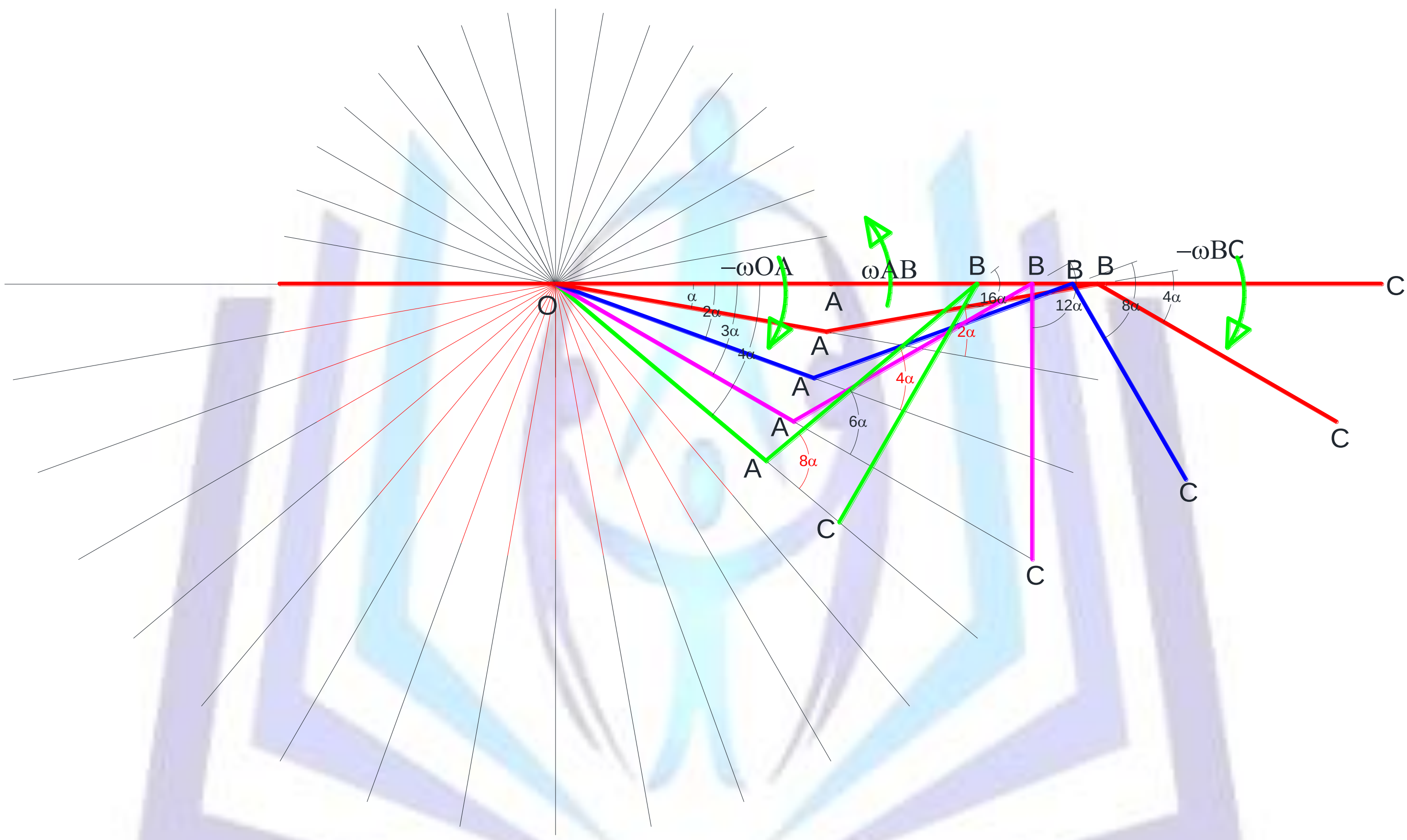
Ex 30

$$\begin{cases} -\boldsymbol{\omega} OA = \dfrac{\pi}{18}\ Rad\ /Sec \\ t = 1\ sec \\ \overline{OA} = \overline{AB} = \overline{BC} \\ -\omega OA = 2.\omega AB = 4.\omega BC \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



Ex 31

$$\begin{cases} \boldsymbol{\omega} OA = \dfrac{-\pi}{18}\ Rad/Sec \\ t = 1\ sec \\ \overline{OA} = \overline{AB} = \overline{BC} \\ -\omega OA = 2.\omega AB = -4.\omega BC \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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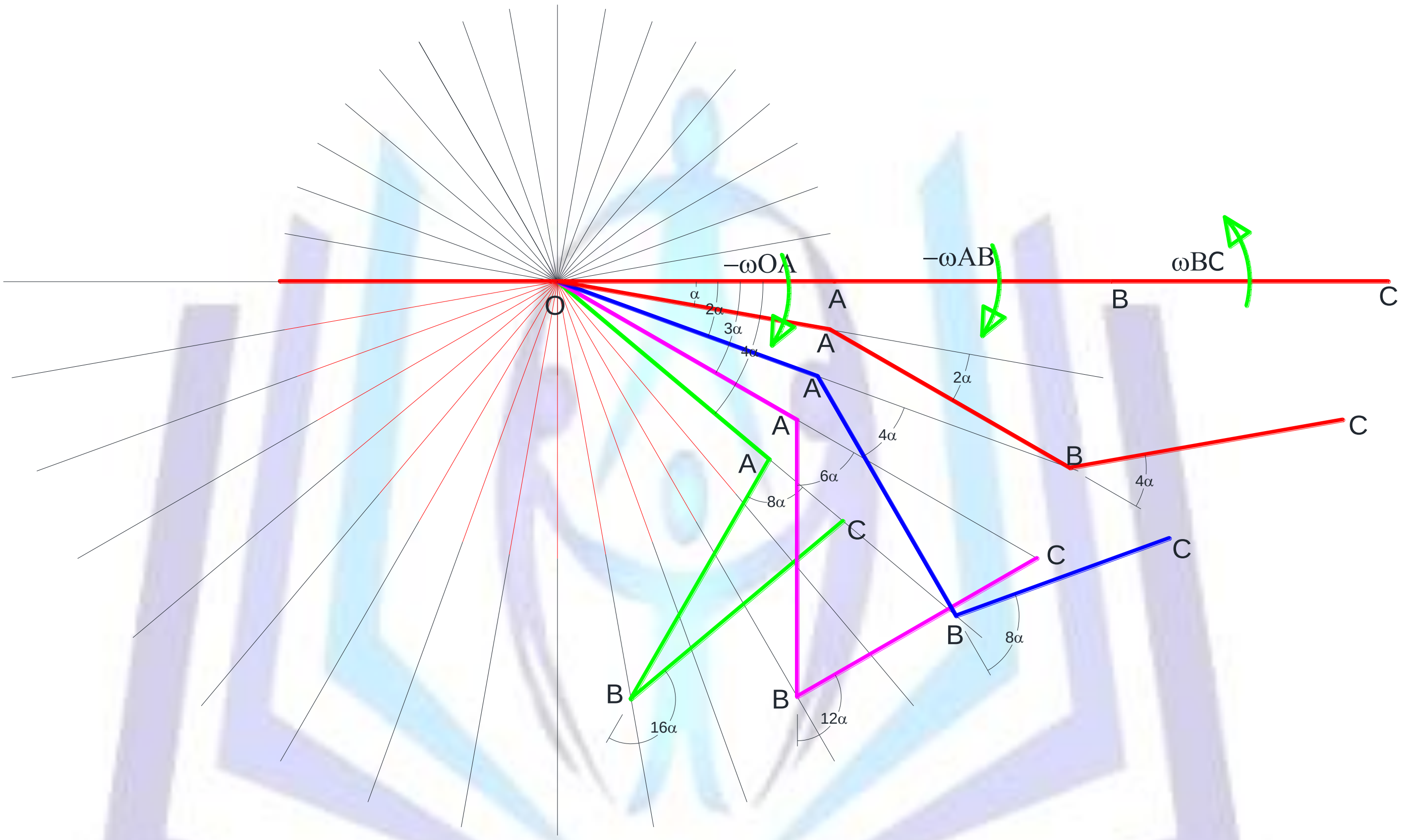
Ex 32

$$\begin{cases} \boldsymbol{\omega} OA = \dfrac{-\pi}{18}\ Rad/Sec \\ t = 1\ sec \\ \overline{OA} = \overline{AB} = \overline{BC} \\ -\omega OA = -2.\omega AB = -4.\omega BC \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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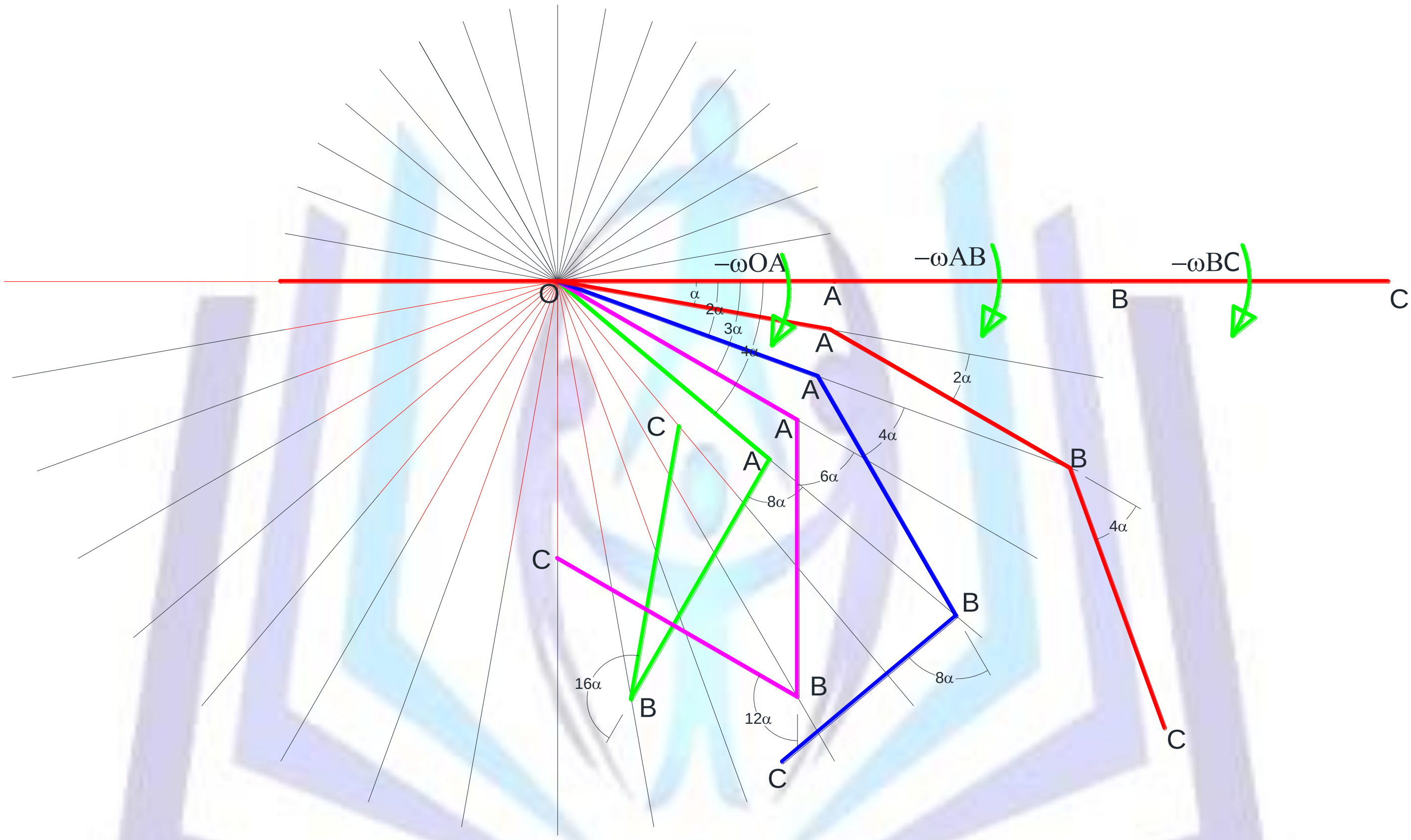
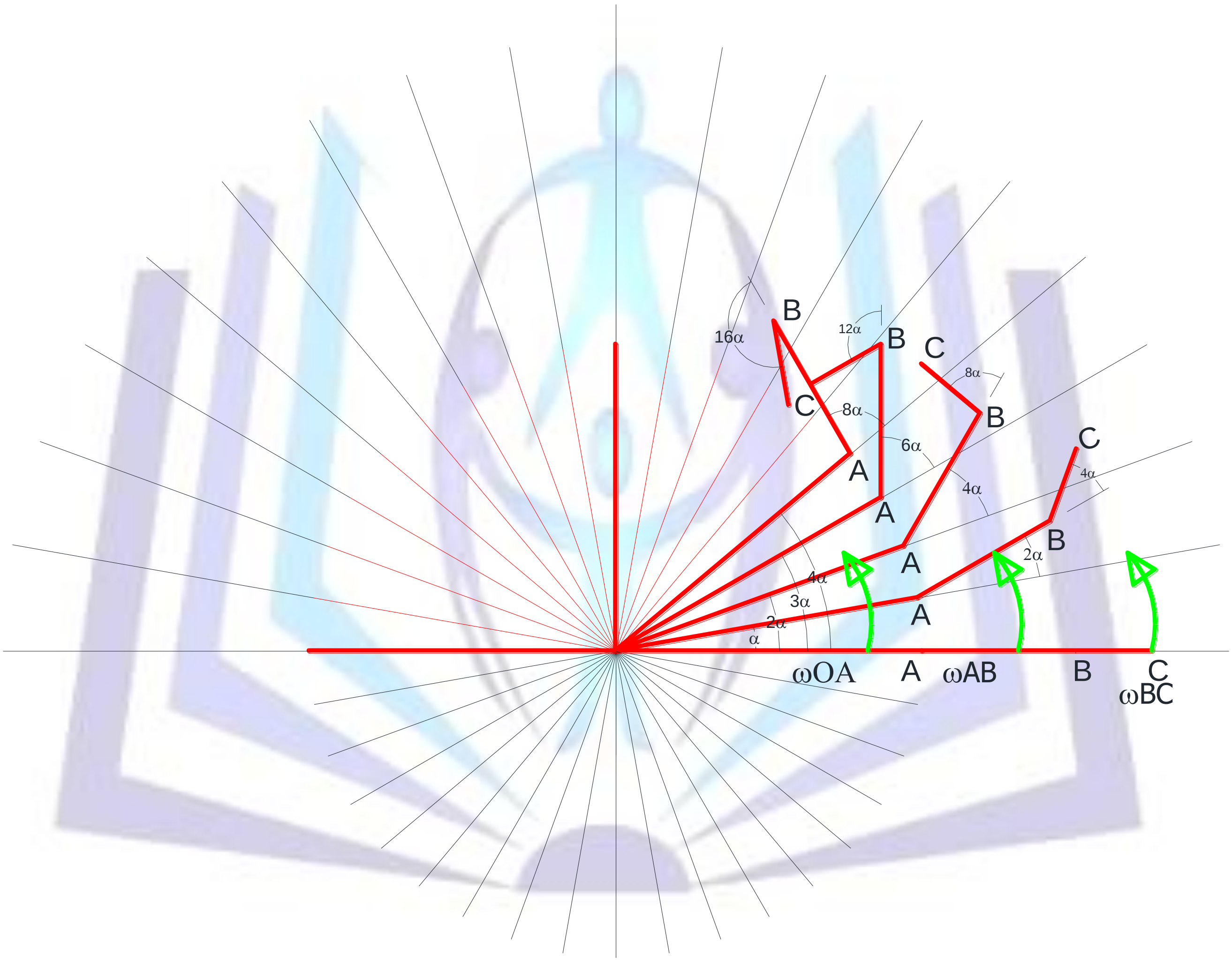
Ex 33

$$\begin{cases} \boldsymbol{\omega} OA = \dfrac{-\pi}{18}\ Rad/Sec \\ t = 1\ sec \\ \overline{OA} = \overline{AB} = \overline{BC} \\ -\omega OA = 2.\omega AB = -4.\omega BC \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



Ex 34

$$\begin{cases} \boldsymbol{\omega OA} = \frac{\pi}{18}\ Rad\ /Sec \\ t = 1\ sec \\ \overline{OA} = \overline{2.AB} = \overline{4.BC} \\ \omega OA = \frac{\omega AB}{2} = \frac{\omega BC}{4} \\ \omega OC = ? \\ \alpha = \frac{\pi}{18} \end{cases}$$



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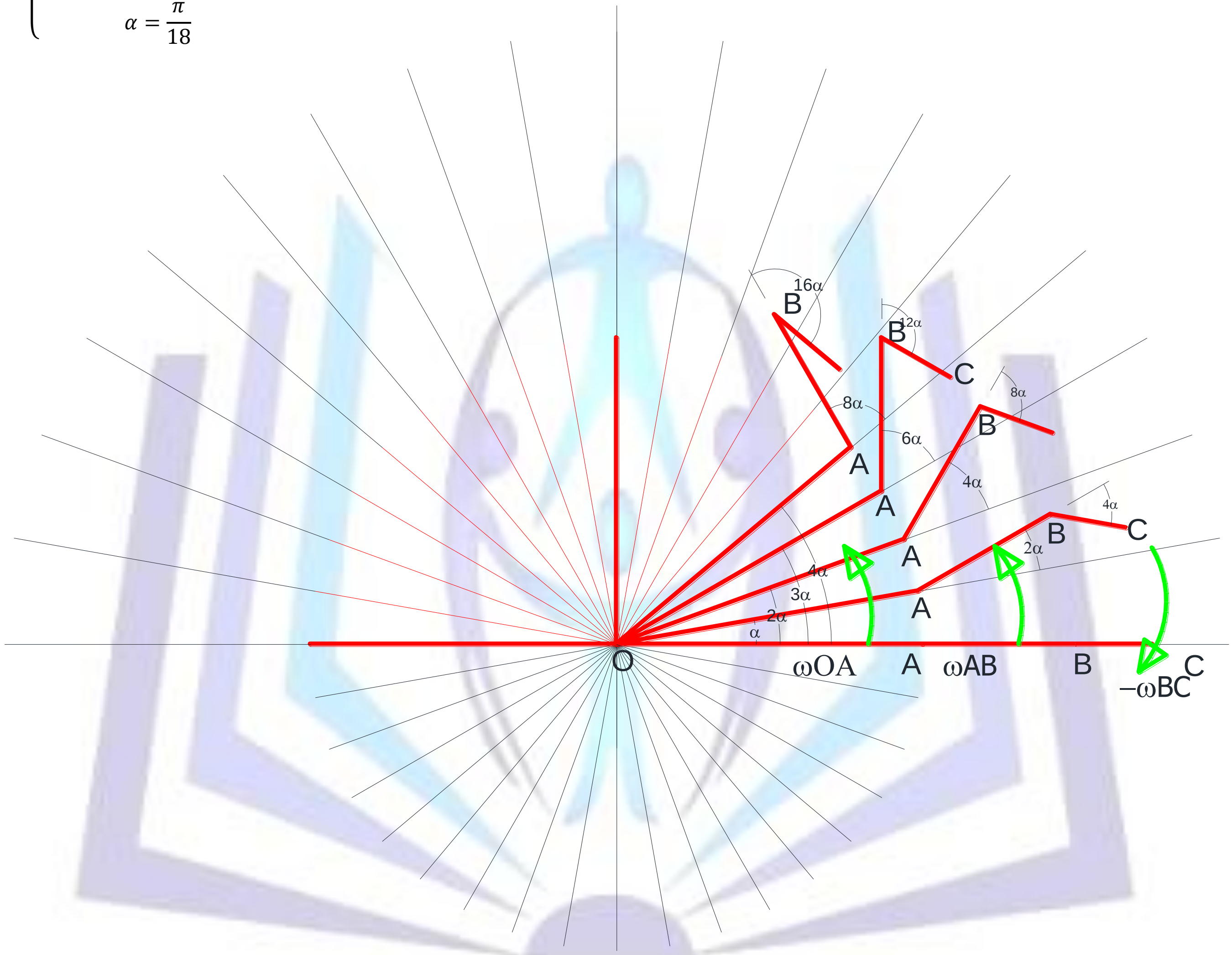
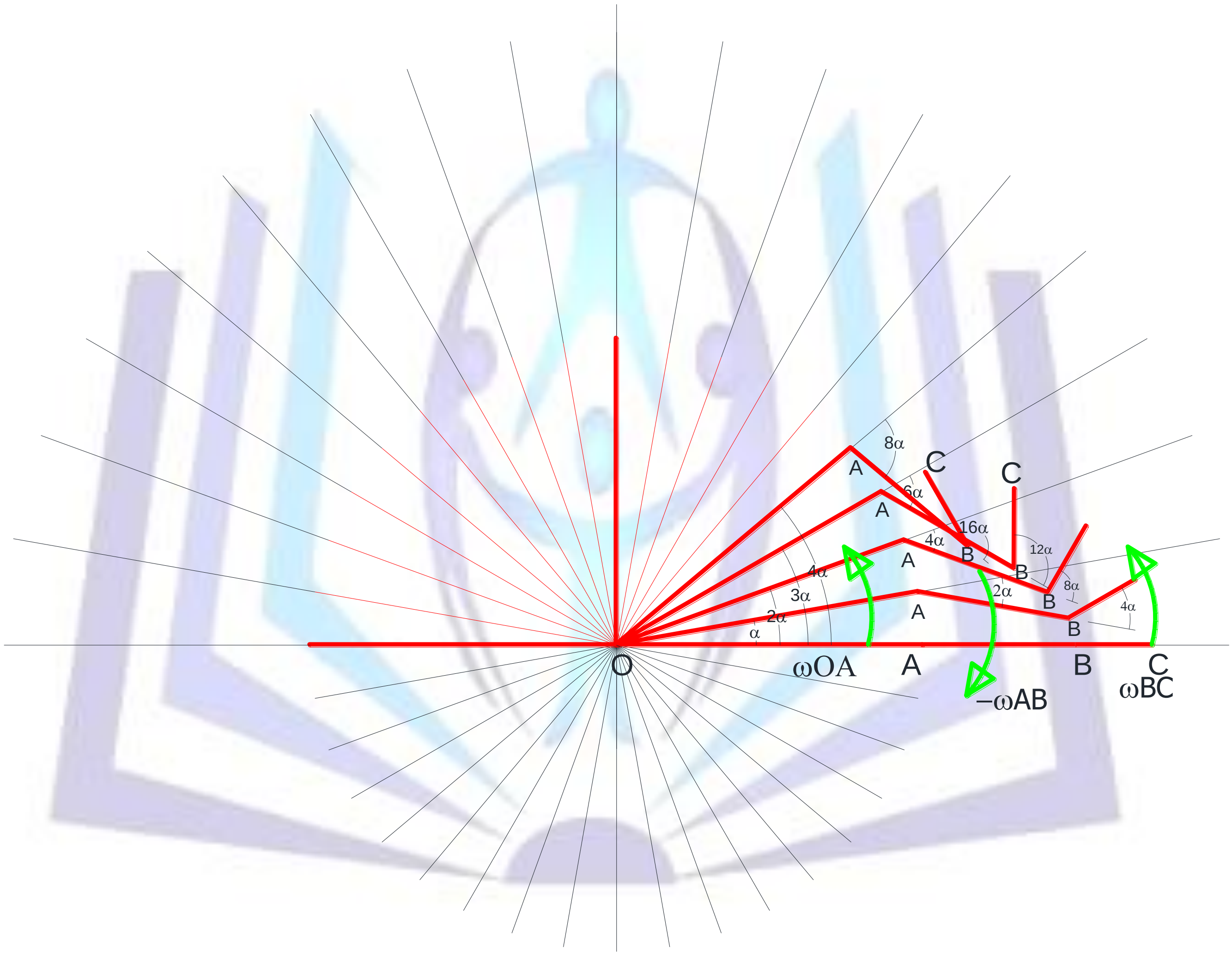
Ex 35

$$\begin{cases} \boldsymbol{\omega} OA = \dfrac{\pi}{18}\ Rad/Sec \\ t = 1\ sec \\ \overline{OA} = \overline{2.AB} = \overline{4.BC} \\ \omega OA = \dfrac{\omega AB}{2} = \dfrac{-\omega BC}{4} \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



Ex 36

$$\begin{cases} \boldsymbol{\omega} OA = \dfrac{\pi}{18}\ Rad\ /Sec \\ t = 1\ sec \\ \overline{OA} = \overline{2.AB} = \overline{4.BC} \\ \omega OA = \dfrac{-\omega AB}{2} = \dfrac{\omega BC}{4} \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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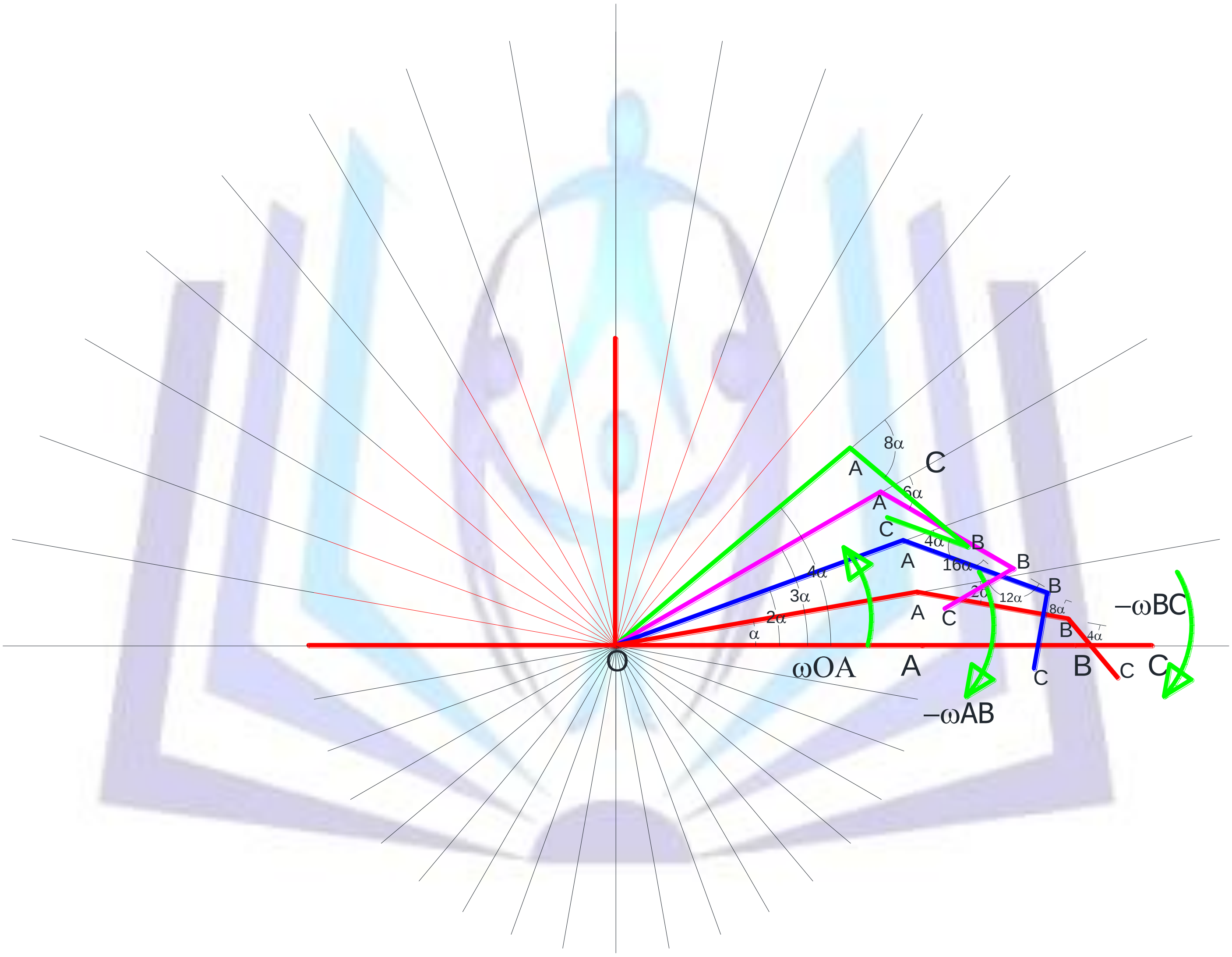
Ex 37

$$\begin{cases} \boldsymbol{\omega} OA = \dfrac{\pi}{18}\ Rad\ /Sec \\ t = 1\ sec \\ \overline{OA} = \overline{2.AB} = \overline{4.BC} \\ \omega OA = \dfrac{-\omega AB}{2} = \dfrac{-\omega BC}{4} \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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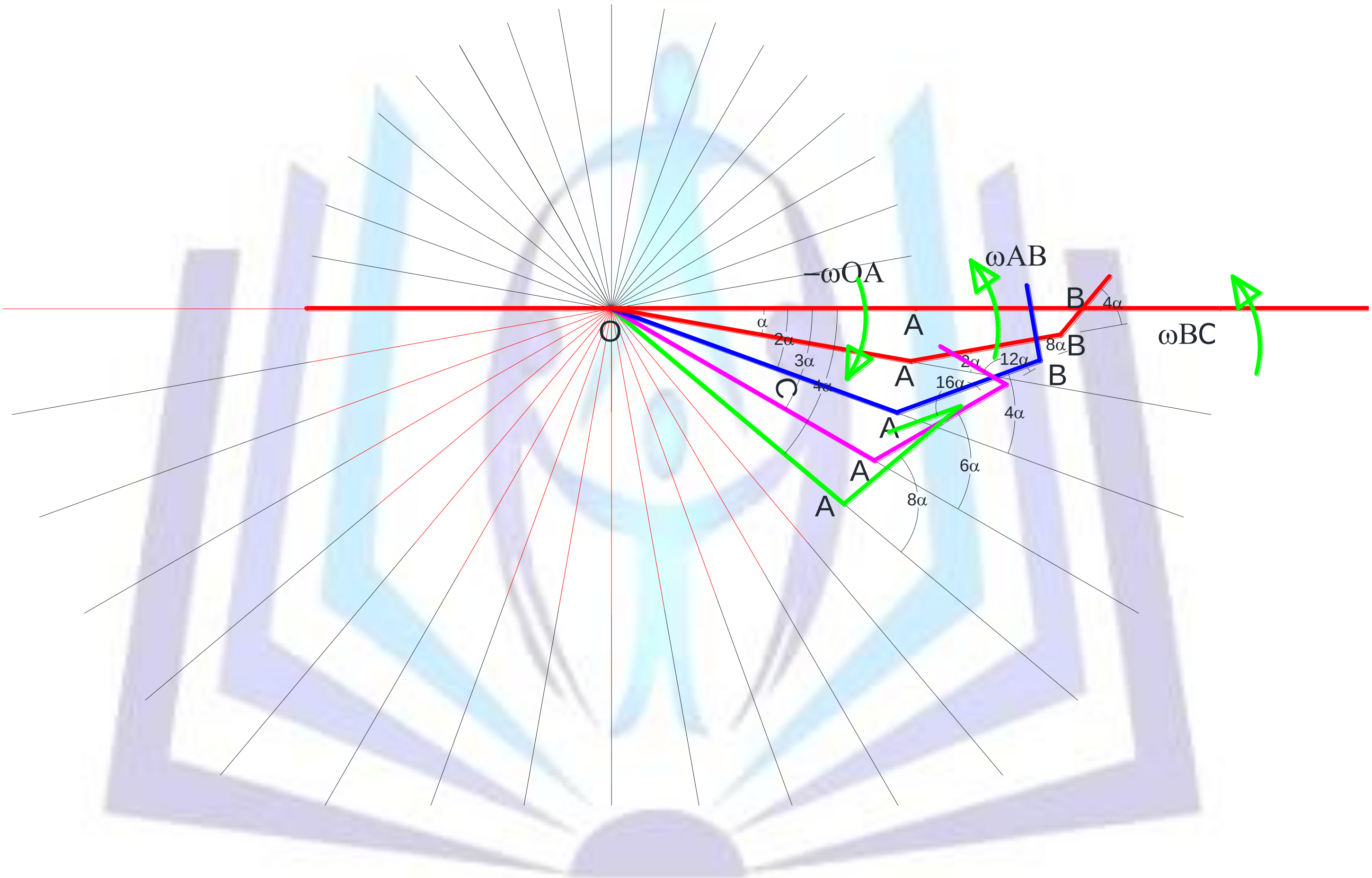
Ex 38

$$\begin{cases} -\boldsymbol{\omega}OA = \dfrac{\pi}{18}\ Rad/Sec \\ \quad t = 1\ sec \\ \overline{OA} = \overline{2.AB} = \overline{4.BC} \\ -\omega OA = \dfrac{\omega AB}{2} = \dfrac{\omega BC}{4} \\ \quad \omega OC = ? \\ \qquad\qquad \alpha = \dfrac{\pi}{18} \end{cases}$$



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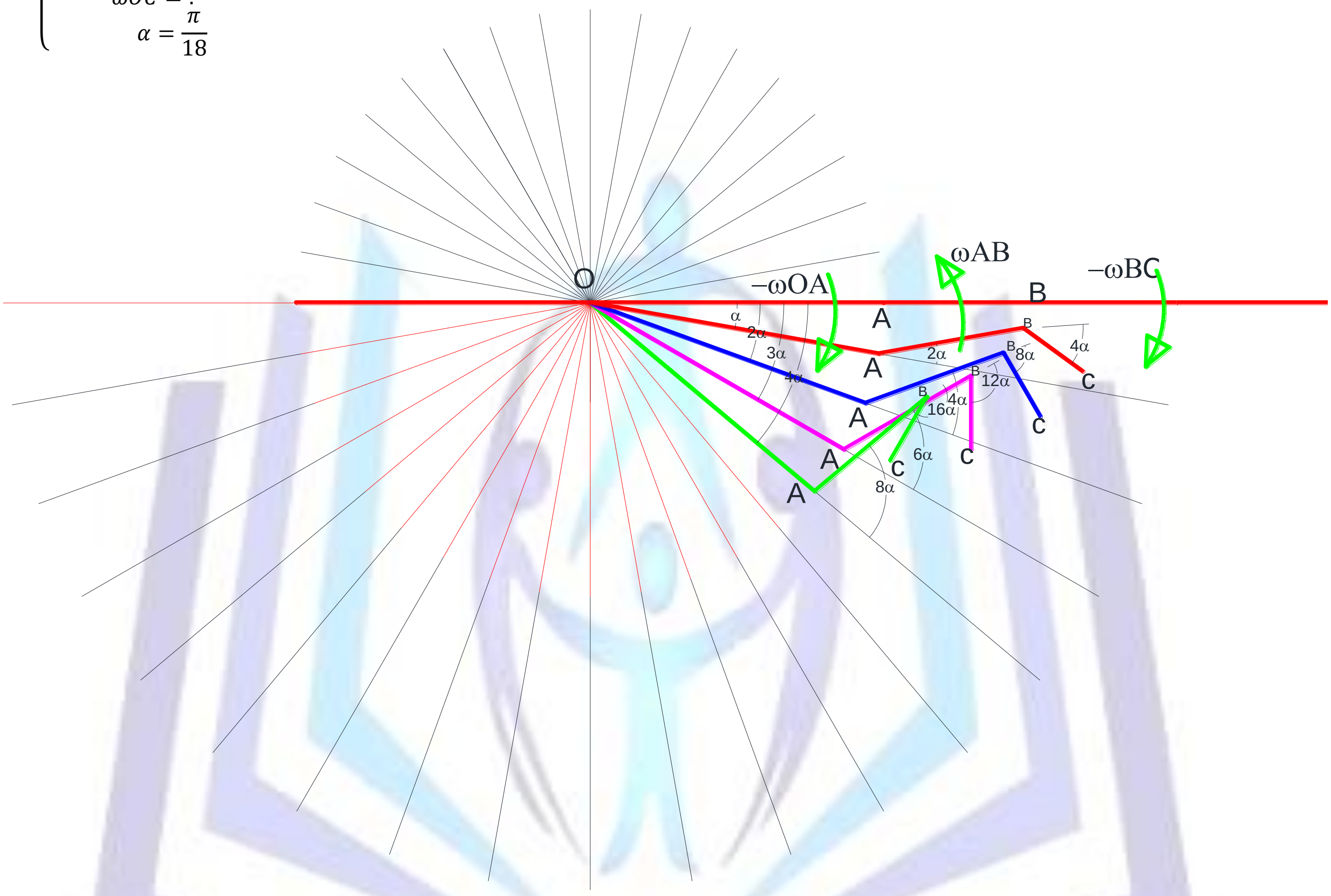
Ex 39

$$\begin{cases} -\boldsymbol{\omega} OA = \dfrac{\pi}{18}\ Rad/Sec \\ t = 1\ sec \\ \overline{OA} = \overline{2.AB} = \overline{4.BC} \\ -\omega OA = \dfrac{\omega AB}{2} = \dfrac{-\omega BC}{4} \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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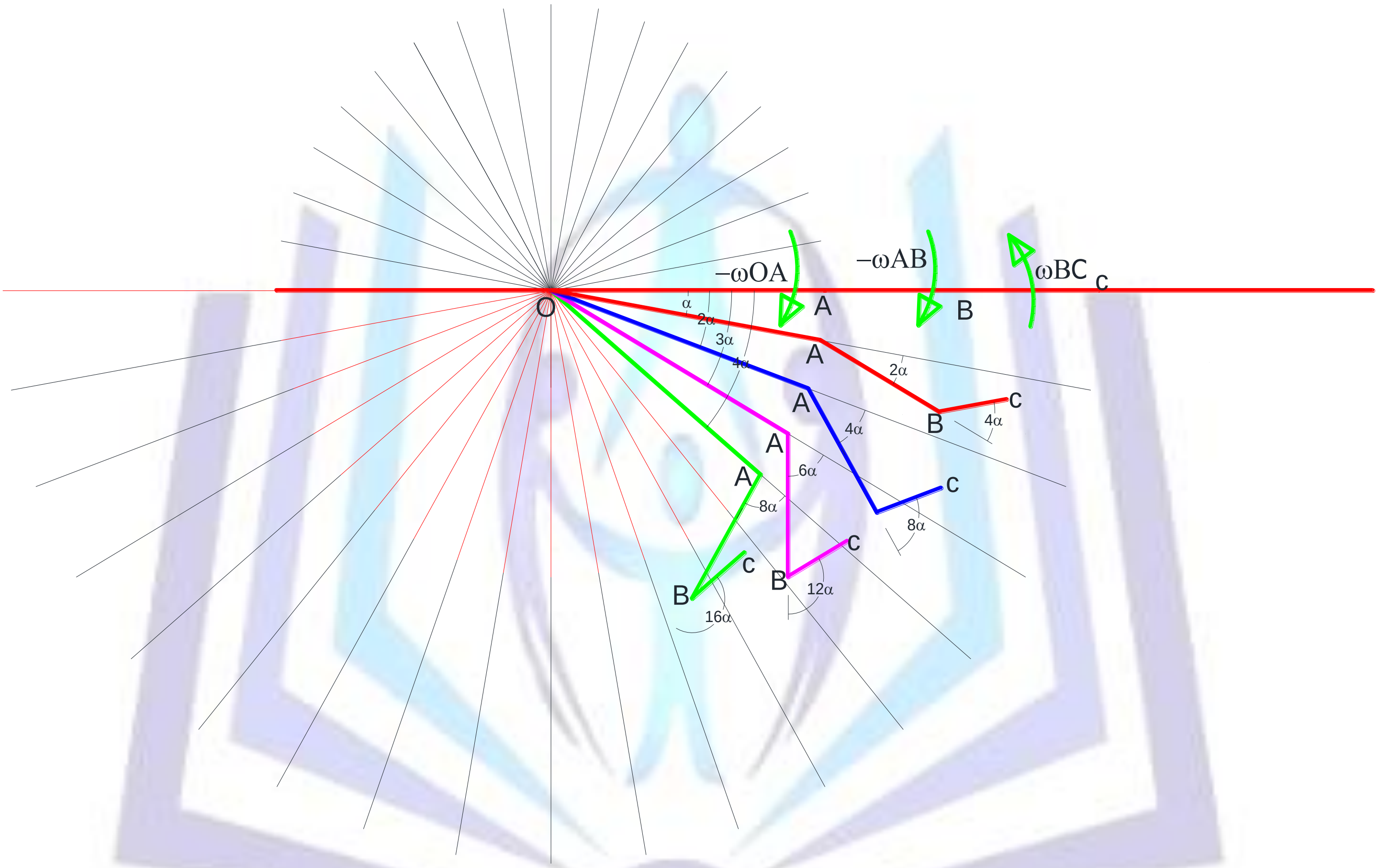
Ex 40

$$\begin{cases} -\boldsymbol{\omega} OA = \dfrac{\pi}{18}\ Rad/Sec \\ t = 1\ sec \\ \overline{OA} = \overline{2.AB} = \overline{4.BC} \\ -\omega OA = \dfrac{-\omega AB}{2} = \dfrac{\omega BC}{4} \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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Ex 41

$$\begin{cases} -\boldsymbol{\omega}OA = \dfrac{\pi}{18}\ Rad/Sec \\ t = 1\ \text{sec} \\ \overline{OA} = \overline{2.AB} = \overline{4.BC} \\ -\omega OA = \dfrac{-\omega AB}{2} = \dfrac{-\omega BC}{4} \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$

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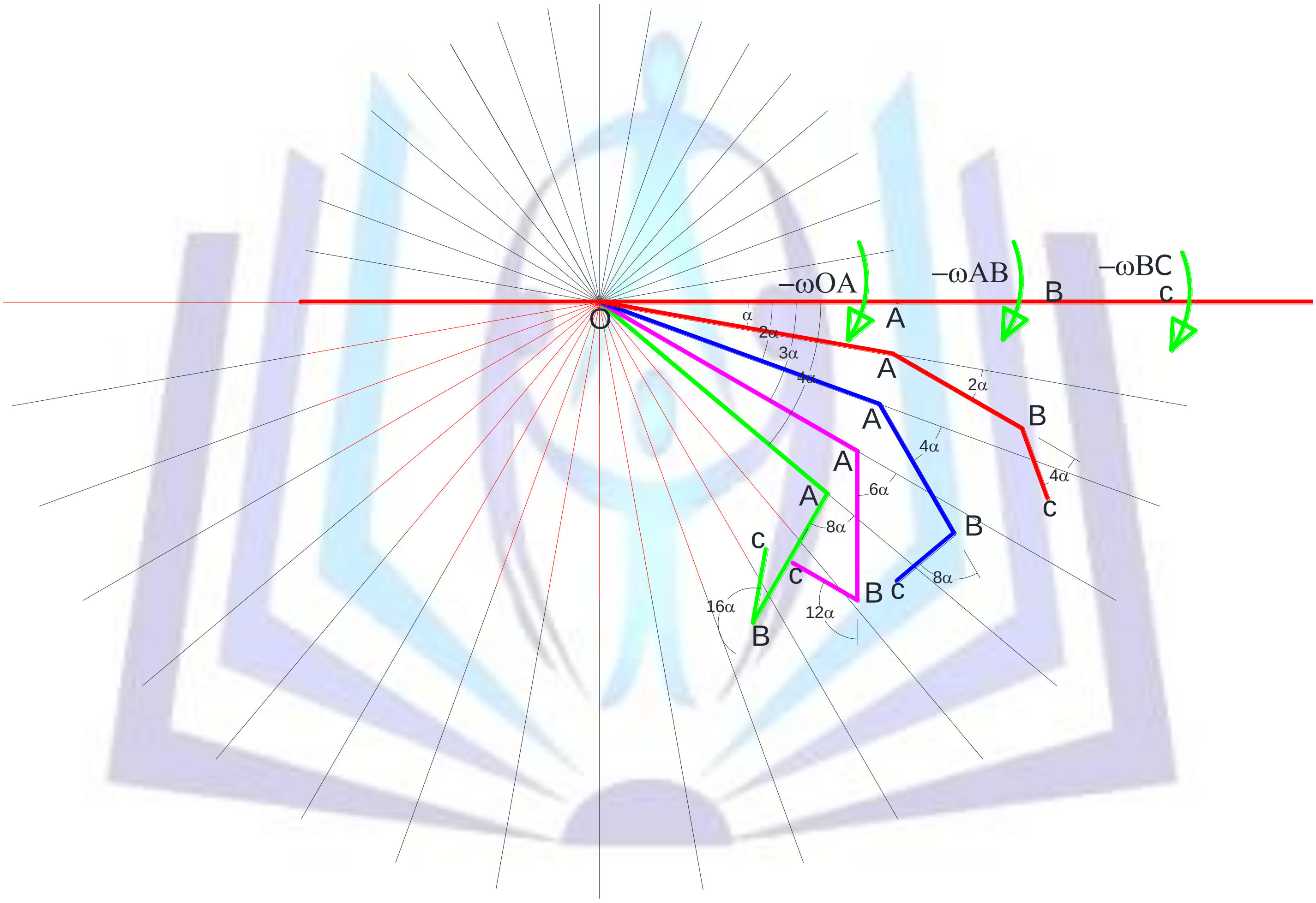

Reference

1- Kippenhahn, R., Weigert, A., & Weiss, A. (2013). The Angular-Velocity Distribution in Stars. In Stellar Structure and Evolution (pp. 575-585). Springer Berlin Heidelberg.
2- Abdul-Razzaq, W., & Golubović, L. (2013). Demonstrating the conservation of angular momentum using model cars moving along a rotating rod. Physics Education, 48(1), 42.
3- Herivel, J. (1965). The background to Newton's Principia (pp. 12-12). Oxford: Clarendon Press.
4- Meriam, J. L., & Kraige, L. G. (2012). Engineering mechanics: dynamics (Vol. 2). John Wiley & Sons Incorporated.
5- Meriam, J. L, Kraige, L. G, Wiley, 2002, Engineering Mechanics: Statics and Engineering Mechanics Dynamics, 5th edition.
6- Beer, Ferdinand P., E. Russell Johnston, William E. Clausen, 2007, Vector Mechanics for Engineers: Statics and Vector Mechanics for Engineers: Dynamics, 8th edition.
7- Pytel, Andrew, Kiusalaas, Jaan, 1999, Engineering Mechanics: Dynamics, 2nd edition

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He fields of Physics , Mechanics, Mathematics has published several articles in Iranian universities and this article and 3 the following article in **JOURNAL OF ADVANCES IN MATHEMATICS** .

1- **Application Of Differential Equations In Spherical Space** (Vol 9, No 9

2- **Moving origins of coordinates** (Vol 10, No 2)

3- **The Resultant formula of Masses m_1/ m_2, ..., m_n in Space oxyz at a Point** (Vol 10, No 5)