



**Physics, Mechanics, Mathematics**

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**IN THE NAME OF GOD**

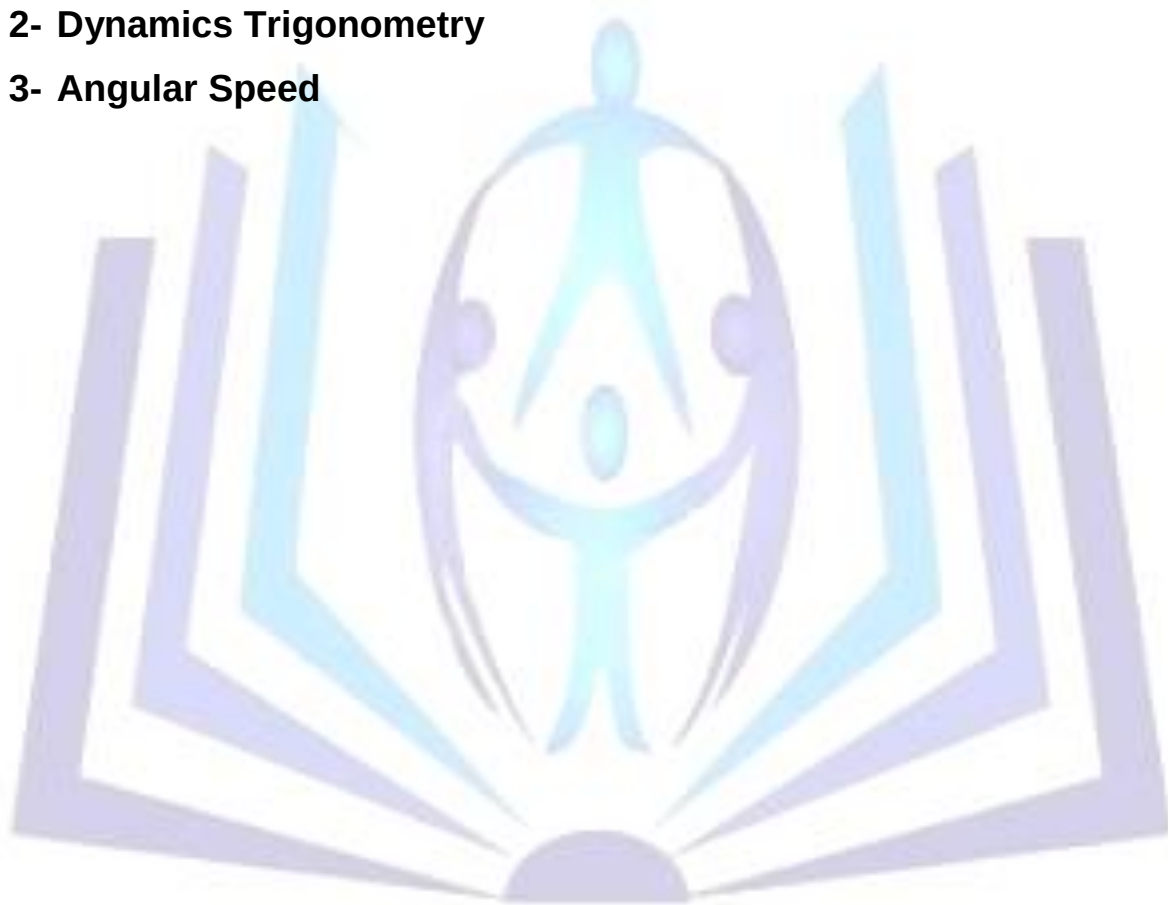
Investigation has been made into 3 issues each studied separately under the following titles:

**Topics**

**1- Stable Dynamics (5th dimension)**

**2- Dynamics Trigonometry**

**3- Angular Speed**



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Now regarding what preceded the proof of non-Euclidean trigonometric circle is dealt with and then, non-Euclidean trigonometric proved formula will be compared with Euclidean trigonometric circle.  
In Euclidean trigonometric circle , we have :

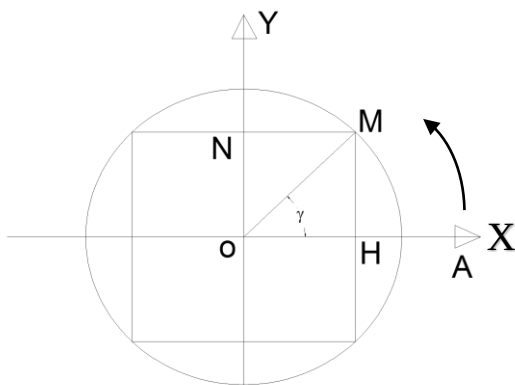


Fig. 1

$$\begin{cases} \overline{OM} = 1 \\ \overline{OH} = \cos \gamma \\ \overline{ON} = \sin \gamma \end{cases}$$

$$(\overline{OH})^2 + (\overline{ON})^2 = (\overline{OM})^2$$

$$\cos^2 \gamma + \sin^2 \gamma = 1$$

Non-Euclidean trigonometric circle is the same as the above Euclidean trigonometric circle (Fig.1) , the difference being that in the above trigonometric circle , the movement of the moving object round axes oy, ox and on the 4 quarters of the trigonometric circle region is studied , so that the movement of the moving object is in the direction of trigonometric movement , that is :

$$\left. \begin{matrix} \overline{OM} = 1 \\ \overline{OH} = (-V_x) \\ \overline{ON} = (V_y) \end{matrix} \right\} \Rightarrow \overline{OH} + \overline{ON} = \overline{OM} \Rightarrow \begin{cases} (-V_x)^2 + (V_y)^2 = 1 \\ V_x^2 + V_y^2 = 1 \end{cases}$$

If  $1 \leq V_y$  and  $1 \leq V_x$  , the following relation can be used:  $C_1 = \text{cte}, (C_1^2 \cdot V_x^2 + C_1^2 \cdot V_y^2 = C_1^2)$

In Euclidean trigonometric circle , the location of each point M is specified on the trigonometric circle.

Euclidean trigonometry can thus be named local trigonometry.

In non-Euclidean trigonometric circle , since the movement of the moving object is studied , and at any moment in time , the moving object has a specific location , non-Euclidean trigonometry can be termed temporal trigonometry.

These 2 local and temporal trigonometric circles are now compared with each other.

In the comparison , it can be concluded that in local trigonometry , only the local situation is followed .

In temporal trigonometry however , both local and temporal situations are followed , and for test and control in 3 planes for local trigonometry,  $(\tan \alpha = \tan \beta \cdot \tan \gamma)$  can be used , and in any type of movement , if temporal trigonometry is applied there will be no need for local trigonometry application . Problems 1 , 2 , ... are only practices . If there are applied problems .

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| First quarter |                                | Second quarter |                                | Third quarter |                                | Fourth quarter |                                |
|---------------|--------------------------------|----------------|--------------------------------|---------------|--------------------------------|----------------|--------------------------------|
| Cos γ         | V <sub>x</sub>                 | Cos γ          | V <sub>x</sub>                 | Cos γ         | V <sub>x</sub>                 | Cos γ          | V <sub>x</sub>                 |
| +             | -                              | -              | -                              | -             | +                              | +              | +                              |
| Sin γ         | V <sub>y</sub>                 | Sin γ          | V <sub>y</sub>                 | Sin γ         | V <sub>y</sub>                 | Sin γ          | V <sub>y</sub>                 |
| +             | +                              | +              | -                              | -             | -                              | -              | +                              |
| tan γ         | V <sub>y</sub> /V <sub>x</sub> | tan γ          | V <sub>y</sub> /V <sub>x</sub> | tan γ         | V <sub>y</sub> /V <sub>x</sub> | tan γ          | V <sub>y</sub> /V <sub>x</sub> |
| +             | -                              | -              | +                              | +             | -                              | -              | +                              |
| Cotan γ       | V <sub>x</sub> /V <sub>y</sub> | Cotan γ        | V <sub>x</sub> /V <sub>y</sub> | Cotan γ       | V <sub>x</sub> /V <sub>y</sub> | Cotan γ        | V <sub>x</sub> /V <sub>y</sub> |
| +             | -                              | -              | +                              | +             | -                              | -              | +                              |

No given relations  $z' = \frac{V_z}{V_x}$  and  $y' = \frac{V_y}{V_x}$  In planes ozy and oxy , the following relations in 3 planes of oxy , oyz and oxz can be written as follows :

1 )  $z' = \frac{V_z}{V_x} = \tan \alpha$  (In plane oxz )

2 )  $y' = \frac{V_y}{V_x} \tan \gamma$  (In plane oxy )

By dividing relation (1) by relation (2) , relation (3) can be obtained in plane oyz :

3 )  $\frac{z'}{y'} = \frac{\frac{V_z}{V_x}}{\frac{V_y}{V_x} \tan \gamma} = \frac{\tan \alpha}{\tan \gamma} = \frac{V_z}{V_y} = \tan \beta \Rightarrow \frac{V_z}{V_y} = \tan \beta$

No , given relations 1 , 2 & 3 , relation 4 is obtained :



$$1) \frac{V_z}{V_x} = \tan \alpha$$

$$2) \frac{V_y}{V_x} = \tan \gamma \quad \frac{V_z}{V_x} = \frac{V_z}{V_y} * \frac{V_y}{V_x} \Rightarrow 4) \tan \alpha = \tan \beta * \tan \gamma$$

$$3) \frac{V_z}{V_y} = \tan \beta$$

Relation (4) can be reckoned the basis for the original formula (dynamic trigonometry) . The following formulas will thus be the subset of formula (4) .

As an example :

$$\cos \alpha = \frac{1}{\sqrt{1+\tan^2 \alpha}} \Rightarrow 5) \cos \alpha = \frac{1}{\sqrt{1+\tan^2 \beta \cdot \tan^2 \gamma}}$$

$$\sin \alpha = \frac{\tan \alpha}{\sqrt{1+\tan^2 \alpha}} = \frac{\tan \beta \cdot \tan \gamma}{\sqrt{1+\tan^2 \beta \cdot \tan^2 \gamma}} \Rightarrow 6) \sin \alpha = \frac{\tan \beta \cdot \tan \gamma}{\sqrt{1+\tan^2 \beta \cdot \tan^2 \gamma}}$$

$$\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha = 2 \frac{\tan \beta \cdot \tan \gamma}{\sqrt{1+\tan^2 \beta \cdot \tan^2 \gamma}} \cdot \frac{1}{\sqrt{1+\tan^2 \beta \cdot \tan^2 \gamma}} = \frac{2 \tan \beta \cdot \tan \gamma}{1+\tan^2 \beta \cdot \tan^2 \gamma}$$

$$7) \sin 2\alpha = \frac{2 \tan \beta \cdot \tan \gamma}{1+\tan^2 \beta \cdot \tan^2 \gamma}$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = \frac{1}{1+\tan^2 \beta \cdot \tan^2 \gamma} - \frac{\tan^2 \beta \cdot \tan^2 \gamma}{1+\tan^2 \beta \cdot \tan^2 \gamma}$$

$$8) \cos 2\alpha = \frac{1 - \tan^2 \beta \cdot \tan^2 \gamma}{1 + \tan^2 \beta \cdot \tan^2 \gamma}$$

$$\tan 2\alpha = \frac{\sin 2\alpha}{\cos 2\alpha} = \frac{\frac{2 \tan \beta \cdot \tan \gamma}{1+\tan^2 \beta \cdot \tan^2 \gamma}}{\frac{1 - \tan^2 \beta \cdot \tan^2 \gamma}{1+\tan^2 \beta \cdot \tan^2 \gamma}} = \frac{2 \tan \beta \cdot \tan \gamma}{1 - \tan^2 \beta \cdot \tan^2 \gamma}$$

$$9) \tan 2\alpha = \frac{2 \tan \beta \cdot \tan \gamma}{1 - \tan^2 \beta \cdot \tan^2 \gamma}$$

### Practice No 1.

Relation (4) is  $\tan \alpha = \tan \beta \cdot \tan \gamma$  Given

$$\frac{\sin \alpha}{\cos \alpha} = \frac{\sin \beta \cdot \sin \gamma}{\cos \beta \cdot \cos \gamma} \Rightarrow \begin{cases} \sin \alpha = \sin \beta \cdot \sin \gamma \\ \cos \alpha = \cos \beta \cdot \cos \gamma \end{cases}$$

, obtain the relation between  $\tan \beta$  ,  $\tan \gamma$  , and in the end , obtain the relations between  $V_x$  ,  $V_y$  and  $V_z$ .

$$\left. \begin{aligned} \sin^2 \alpha &= \sin^2 \beta \cdot \sin^2 \gamma \\ \cos^2 \alpha &= \cos^2 \beta \cdot \cos^2 \gamma \end{aligned} \right\} \Rightarrow \sin^2 \alpha + \cos^2 \alpha = 1 = \sin^2 \beta \cdot \sin^2 \gamma + \cos^2 \beta \cdot \cos^2 \gamma$$

$$\begin{aligned} \cos \beta &= \frac{1}{\sqrt{1+\tan^2 \beta}} & \cos \gamma &= \frac{1}{\sqrt{1+\tan^2 \gamma}} \\ \sin \beta &= \frac{\tan \beta}{\sqrt{1+\tan^2 \beta}} & \sin \gamma &= \frac{\tan \gamma}{\sqrt{1+\tan^2 \gamma}} \end{aligned}$$

$$1 = \sin^2 \beta \cdot \sin^2 \gamma + \cos^2 \beta \cdot \cos^2 \gamma$$

If we substitute values  $\sin \beta$  ,  $\cos \beta$  ,  $\sin \gamma$  ,  $\cos \gamma$  in terms of  $\tan \gamma$  and  $\tan \beta$  in the above relation we will have :

$$\frac{1}{1} = \frac{\tan^2 \beta}{(1 + \tan^2 \beta)} \cdot \frac{\tan^2 \gamma}{(1 + \tan^2 \gamma)} + \frac{1}{(1 + \tan^2 \beta) \cdot (1 + \tan^2 \gamma)} \Rightarrow (1 + \tan^2 \beta \cdot \tan^2 \gamma) = (1 + \tan^2 \beta) \cdot (1 + \tan^2 \gamma)$$

$$1 + \tan^2 \gamma + \tan^2 \beta + \tan^2 \beta \cdot \tan^2 \gamma = 1 + \tan^2 \beta \cdot \tan^2 \gamma$$

$$\tan^2 \gamma + \tan^2 \beta = 0 \Rightarrow \frac{V_y^2}{V_x^2} + \frac{V_z^2}{V_y^2} = 0$$

### Practice No 2.

Given  $\tan \alpha = \tan \beta \cdot \tan \gamma \Rightarrow \tan \gamma = \frac{\tan \alpha}{\tan \beta}$  , calculate the relations between  $V_x$  ,  $V_y$  &  $V_z$

$$\left. \begin{aligned} \tan \alpha &= \frac{V_z}{V_x} \\ \tan \beta &= \frac{V_z}{V_y} \\ \tan \gamma &= \frac{V_y}{V_x} \end{aligned} \right\} \Rightarrow \tan \gamma = \frac{\tan \alpha}{\tan \beta} \Rightarrow \frac{\sin \gamma}{\cos \gamma} = \frac{\sin \alpha / \cos \alpha}{\sin \beta / \cos \beta} = \frac{\sin \alpha \cdot \cos \beta}{\cos \alpha \cdot \sin \beta}$$



$$\begin{cases} \sin \gamma = \sin \alpha \cdot \cos \beta \Rightarrow \sin^2 \gamma = \sin^2 \alpha \cdot \cos^2 \beta \\ \cos \gamma = \cos \alpha \cdot \sin \beta \Rightarrow \cos^2 \gamma = \cos^2 \alpha \cdot \sin^2 \beta \end{cases}$$

$$\sin^2 \gamma + \cos^2 \gamma = 1 \Rightarrow \sin^2 \gamma + \cos^2 \gamma = \sin^2 \alpha \cdot \cos^2 \beta + \cos^2 \alpha \cdot \sin^2 \beta = 1$$

$$\sin \alpha = \frac{\tan \alpha}{\sqrt{1+\tan^2 \alpha}} = \frac{V_z/V_x}{\sqrt{1+(\frac{V_z}{V_x})^2}}, \quad \sin \beta = \frac{\tan \beta}{\sqrt{1+\tan^2 \beta}} = \frac{V_z/V_y}{\sqrt{1+(\frac{V_z}{V_y})^2}}$$

$$\cos \alpha = \frac{1}{\sqrt{1+\tan^2 \alpha}} = \frac{1}{\sqrt{1+(\frac{V_z}{V_x})^2}}, \quad \cos \beta = \frac{1}{\sqrt{1+\tan^2 \beta}} = \frac{1}{\sqrt{1+(\frac{V_z}{V_y})^2}}$$

$$\frac{(\frac{V_z}{V_x})^2}{1+(\frac{V_z}{V_x})^2} \cdot \frac{1}{1+(\frac{V_z}{V_y})^2} + \frac{1}{1+(\frac{V_z}{V_x})^2} \cdot \frac{(\frac{V_z}{V_y})^2}{1+(\frac{V_z}{V_y})^2} = \frac{1}{1}$$

$$\frac{(\frac{V_z}{V_x})^2 + (\frac{V_z}{V_y})^2}{[1+(\frac{V_z}{V_x})^2] \cdot [1+(\frac{V_z}{V_y})^2]} = \frac{1}{1} \Rightarrow (\frac{V_z}{V_x})^2 + (\frac{V_z}{V_y})^2 = 1 + (\frac{V_z}{V_y})^2 + (\frac{V_z}{V_x})^2 + (\frac{V_z}{V_x})^2 \cdot (\frac{V_z}{V_y})^2$$

$$1 + \frac{V_z^2}{V_x^2} \cdot \frac{V_z^2}{V_y^2} = 0 \Rightarrow V_z^4 = -V_x^2 \cdot V_y^2$$