## Optical trigonometry

We know that the light source is radiated and emitted in all directions. The speed of each ray of light is 300,000 kilometers per hour. The following options can be calculated to check for a light ray or more than one light:

1- The propagation of a ray of light along one of the axes of the coordinates (ox), (oy), (oz)
2- Images of the emission of a light ray on the oxy coordinate plane
3- Images of a ray of light emitting in the coordinate axis (oxyz)

1. Emission of a ray of light along one of the coordinate axes (ox, oy, oz): Given the constant speed of light, we can write:
$V_{X}=V_{x}$
$\left.V_{Y}=V_{y}\right\}=300,000 \mathrm{~km} / \mathrm{sec}$
$V_{z}=V_{z}$
2. Images of the emission of a ray of light on the oxy coordinate plane: Given the constant speed of light, we can write:
$\mathrm{V}_{\mathrm{X}}{ }^{2}+\mathrm{V}_{\mathrm{Y}}{ }^{2}=(300,000 \mathrm{~km} / \mathrm{sec})^{2}$
3. Images of a ray of light emitting in the axis of space (oxyz). Given the constant speed of light, we can write:
$\mathrm{V}_{\mathrm{x}}{ }^{2}+\mathrm{V}_{Y}{ }^{2}+\mathrm{V}_{3}{ }^{2}=(300,000 \mathrm{~km} / \mathrm{sec})^{2}$

## 1-2- On the (oxy) coordinate plate:

As each ray is propagated linearly along $\sqrt{x^{2}+y^{2}}$, the formula for the speed of light propagation along $\sqrt{x^{2}+y^{2}}$ must also be calculated. Therefore, in the case of the spacecraft velocity equation (oxyz), the light emission rate formula is calculated linearly. (On the oxy coordinate plate) and then formulated the light emission velocity equal to $300,000 \mathrm{~km} / \mathrm{h}$.
$V_{s}=\frac{x \cdot V_{x}+y \cdot V_{y}}{\sqrt{x^{2}+y^{2}}}=300,000 \mathrm{~km} / \mathrm{sec}$
We will have $V_{s}$ images on two axes:

$$
\left.\begin{array}{l}
\mathrm{V}_{\mathrm{X}}=\mathrm{V}_{\mathrm{s}} \cdot \operatorname{Cos}_{\alpha} \\
\mathrm{V}_{\mathrm{Y}}=\mathrm{V}_{\mathrm{S}} \cdot \operatorname{Sin}_{\alpha}
\end{array}\right\} \Rightarrow \tan _{\alpha}=\frac{\mathrm{V}_{\mathrm{Y}}}{\mathrm{VX}}=\frac{y}{x}\left\{\begin{array}{l}
\operatorname{Cos}_{\alpha}=\frac{1}{\sqrt{1+\tan \alpha^{2}}}=\frac{1}{\sqrt{1+\frac{y^{2}}{x^{2}}}} \\
\operatorname{Sin}_{\alpha}=\frac{\tan _{\alpha}}{\sqrt{1+\tan \alpha^{2}}}=\frac{\frac{y}{x}}{\sqrt{1+\frac{y^{2}}{x^{2}}}} \\
\mathrm{~V}_{\mathrm{X}}=\mathrm{V}_{\mathrm{s}} \cdot \operatorname{Cos}_{\alpha}=\frac{x \cdot V_{x}+y \cdot V_{y}}{\sqrt{x^{2}+y^{2}}} \cdot \frac{x}{\sqrt{x^{2}+y^{2}}}=\frac{x\left(x \cdot V_{x}+y \cdot V_{y}\right)}{x^{2}+y^{2}} \\
\mathrm{~V}_{\mathrm{Y}}=\mathrm{V}_{\mathrm{s}} \cdot \operatorname{Sin}_{\alpha}=\frac{x \cdot V_{x}+y \cdot V_{y}}{\sqrt{x^{2}+y^{2}}} \cdot \frac{y}{\sqrt{x^{2}+y^{2}}}=\frac{y\left(x \cdot V_{x}+y \cdot V_{y}\right)}{x^{2}+y^{2}} \\
\boldsymbol{V}_{s}=\frac{\boldsymbol{x} \cdot \boldsymbol{V}_{\boldsymbol{x}}+\boldsymbol{y} \cdot \boldsymbol{V}_{\boldsymbol{y}}}{\sqrt{\boldsymbol{x}^{2}+\boldsymbol{y}^{2}}}=\mathbf{3 0 0 , 0 0 0} \mathbf{~ k m} / \mathbf{s e c}
\end{array}\right.
$$

$$
\begin{array}{r}
\widehat{\mathrm{ABC}} \Rightarrow\left\{\begin{array}{l}
\overrightarrow{\mathrm{AC}}=V_{x} \\
\overrightarrow{\mathrm{CB}}=V_{y}
\end{array}\right. \\
\widehat{\mathrm{AEH}} \Rightarrow\left\{\begin{array}{l}
\overrightarrow{\mathrm{AH}}=V_{x} \\
\overrightarrow{\mathrm{HE}}=V_{Y}
\end{array}\right. \\
\widehat{\mathrm{ABC}} \Rightarrow \frac{\mathrm{~V}_{\mathrm{CB}}}{\mathrm{~V}_{\mathrm{AC}}}=\frac{\mathrm{V}_{\mathrm{y}}}{\mathrm{~V}_{\mathrm{x}}}=\tan \theta
\end{array} \quad \begin{array}{r}
\widehat{\mathrm{AEH}} \Rightarrow \frac{\overline{\mathrm{HE}}}{\mathrm{AH}}=\tan \alpha=\frac{Y}{\mathrm{x}} \\
\widehat{\mathrm{ABE}} \Rightarrow \mathrm{~V}_{\mathrm{AE}}=\mathrm{V}_{\mathrm{AB}} \cdot \cos (\theta-\alpha)
\end{array}
$$

$\cos (\theta-\alpha)=\cos \alpha \cdot \cos \theta+\sin \alpha \cdot \sin \theta$
$\cos \alpha=\frac{1}{\sqrt{1+\tan ^{2} \alpha}}=\frac{1}{\sqrt{1+\frac{y^{2}}{x^{2}}}}=\frac{x}{\sqrt{x^{2}+y^{2}}}$
$\sin \alpha=\frac{\tan \alpha}{\sqrt{1+\tan ^{2} \alpha}}=\frac{y / x}{\sqrt{1+\frac{y^{2}}{x^{2}}}}=\frac{y}{\sqrt{x^{2}+y^{2}}}$
$\cos \theta=\frac{1}{\sqrt{1+\tan ^{2} \theta}}=\frac{1}{\sqrt{1+\frac{V^{2} y}{V^{2} x}}}=\frac{V_{x}}{\sqrt{V^{2} x+V^{2} y}}$
$\sin \theta=\frac{\tan \theta}{\sqrt{1+\tan ^{2} \alpha}}=\frac{V_{y} / V_{x}}{\sqrt{1+\frac{V^{2} y}{V^{2} x}}}=\frac{V_{y}}{\sqrt{V^{2}{ }_{x}+V^{2}{ }_{y}}}$
$\cos (\theta-\alpha)=\frac{x}{\sqrt{x^{2}+y^{2}}} \cdot \frac{V_{x}}{\sqrt{V^{2} x+V^{2} y}}+\frac{y}{\sqrt{x^{2}+y^{2}}} \cdot \frac{V_{y}}{\sqrt{V^{2} x+V^{2} y}}=\frac{x \cdot V_{x}+y \cdot V_{y}}{\sqrt{x^{2}+y^{2}} \cdot \sqrt{V^{2} x+V^{2} y}} \Rightarrow\left\{\begin{array}{l}V_{A B}=\sqrt{V^{2} x+V^{2} y} \\ V_{A E}=V_{A B} \cdot \cos (\theta-\alpha)\end{array}\right.$
$V_{A E}=\sqrt{V^{2}{ }_{x}+V^{2}{ }_{y}} \cdot \frac{x \cdot V_{x}+y \cdot V_{y}}{\sqrt{x^{2}+y^{2}} \cdot \sqrt{V^{2}{ }_{x}+V^{2}{ }_{y}}}=\frac{x \cdot V_{x}+y \cdot V_{y}}{\sqrt{x^{2}+y^{2}}}$
$V_{A H}=V_{\mathrm{x}}=V_{A E} \cdot \cos \alpha=\frac{x \cdot V_{x}+y \cdot V_{y}}{\sqrt{x^{2}+y^{2}}} \cdot \frac{x}{\sqrt{x^{2}+y^{2}}}=\frac{x\left(x \cdot V_{x}+y \cdot V_{y}\right)}{x^{2}+y^{2}}$
$V_{H E}=V_{Y}=V_{A E} \cdot \sin \alpha=\frac{x \cdot V_{x}+y \cdot V_{y}}{\sqrt{x^{2}+y^{2}}} \cdot \frac{y}{\sqrt{x^{2}+y^{2}}}=\frac{y\left(x \cdot V_{x}+y \cdot V_{y}\right)}{x^{2}+y^{2}}$

$(O X Z)\left\{\begin{array}{l|l}\left\{\begin{array}{l}\mathrm{x} \\ \mathrm{z}\end{array} \Rightarrow\left\{\begin{array}{l}\mathrm{V}_{\mathrm{x}}=\frac{x\left(\mathrm{x} \cdot \mathrm{V}_{\mathrm{x}}+\mathrm{z} \cdot \mathrm{V}_{\mathrm{z}}\right)}{\left(\mathrm{x}^{2}+\mathrm{z}^{2}\right)} \\ \mathrm{V}_{\mathrm{z}}=\frac{\mathrm{z}\left(\mathrm{x} \cdot \mathrm{V}_{\mathrm{x}}+\mathrm{z} \cdot \mathrm{V}_{\mathrm{z}}\right)}{\left(x^{2}+\mathrm{z}^{2}\right)}\end{array}\right\}\right.\end{array}\right\}$
$(O X Y Z) \Rightarrow V_{T}=\sqrt{V^{2}{ }_{x}+V^{2}{ }_{y}+V^{2}{ }_{z}}$

$$
0<\mathrm{V}_{\mathrm{T}} \leq 300.000 \mathrm{~km} / \mathrm{sec}
$$

1-3- In (oxyz) space: In 3D space (oxyz) the velocity ( $\mathrm{V}_{\mathrm{s}}$ ) will be equal to ( $300,000 \mathrm{~km} / \mathrm{sec}$ ) and will have 3 images on the coordinate axes (ox, oy, oz).
$V_{s}=300,00 \mathrm{~km} / \mathrm{sec}$
$\left.\begin{array}{l}V_{x}=\frac{x\left(x \cdot V_{x}+y \cdot V_{y}\right)}{x^{2}+y^{2}} \\ V_{Y}=\frac{y\left(x \cdot V_{x}+y \cdot V_{y}\right)}{x^{2}+y^{2}} \\ V_{z}=\frac{z\left(x \cdot V_{x}+z \cdot V_{z}\right)}{x^{2}+z^{2}}\end{array}\right\} \Rightarrow V_{s}=\sqrt{V^{2}{ }_{x}+V^{2}{ }_{y}+V^{2}{ }_{3}=300,00 \mathrm{~km} / \mathrm{sec}}$

Therefore, with the stability of the above, we will have the basis of the basic relationships in optical trigonometry:
$(o x y)\left\{\begin{array}{l}\operatorname{Sin} \alpha=\frac{y}{\sqrt{x^{2}+y^{2}}} \\ \operatorname{Cos} \alpha=\frac{x}{\sqrt{x^{2}+y^{2}}} \\ \operatorname{Tan} \alpha=\frac{y}{x}\end{array}\right.$
$(o x z)\left\{\begin{array}{l}\operatorname{Sin} \beta=\frac{z}{\sqrt{x^{2}+y^{2}}} \\ \operatorname{Cos} \beta=\frac{x}{\sqrt{x^{2}+y^{2}}} \\ \operatorname{Tan} \beta=\frac{z}{x}\end{array}\right.$
$(o y z)\left\{\begin{array}{l}\operatorname{Sin} \gamma=\frac{z}{\sqrt{y^{2}+z^{2}}} \\ \operatorname{Cos} \gamma=\frac{y}{\sqrt{y^{2}+z^{2}}} \\ \operatorname{Tan} \gamma=\frac{z}{y}\end{array}\right.$
Given the proof of the initial relations in optical trigonometry, we can prove the relation of convex trigonometry to optical trigonometry: In convex trigonometry the relations ( $\mathrm{x}, \mathrm{y}, \mathrm{z}$ ) are defined by the following relation:

$$
\begin{equation*}
\left(\frac{z}{x}=\frac{z}{y} \cdot \frac{y}{x}\right) \tag{3-1}
\end{equation*}
$$

Convex triangles are defined by the relationships between ( $x, y, z$ ) on the surface of the sphere or the surface of any curve in space. Images of each point on each curve in space are defined by 3 images in 3 coordinate source plates.

On page (oxy) $\Rightarrow \frac{y}{x}=\tan \alpha$, on page (oxz) $\Rightarrow \frac{z}{x}=\tan \beta$ and on page (oyz) $\Rightarrow \frac{z}{y}=\tan \gamma$ and based on the three relationships, Relationship (3-1) is defined:

$$
\begin{equation*}
\frac{z}{x}=\frac{z}{y} \cdot \frac{y}{x} \Rightarrow(\tan \beta=\tan \gamma \cdot \tan \alpha) \tag{3-2}
\end{equation*}
$$

We can now consider the relationship (3-2) with the basic relations of optical trigonometry.
Now, by proving the relationships of optical trigonometry with convex trigonometry, we now turn to proving dynamic trigonometry with optical trigonometry and convex trigonometry.

In dynamic trigonometry the relationships between $\left(V_{x}, V_{y}, V_{z}\right)$ are defined by the following relation:

$$
\begin{equation*}
\left(\frac{V_{z}}{V_{x}}=\frac{V_{z}}{V_{y}} .\right) \tag{4-1}
\end{equation*}
$$

(oxy) $\Rightarrow \frac{V_{y}}{V_{x}}=\tan \theta_{x y}$
(oxz) $\Rightarrow \frac{V_{z}}{V_{x}}=\tan \theta_{x z}$
(oyz) $\Rightarrow \frac{V_{z}}{V_{y}}=\tan \theta_{y z}$
The proof of the above relationships can be found on the site (www.p3m.ir) under the heading "Dynamic Trigonometry".

Given the proof of the above relationships in multiple trigonometry, optical trigonometry can be seen as the basic trigonometry in the world of physics, mechanics and modern mathematics.

## Result:

In physics, mechanics, mathematics (P3M) we have several types of trigonometry:
1- Euclidean Triangle (Pythagoras):

$$
\begin{aligned}
& \operatorname{Sin}^{2} \alpha+\operatorname{Cos}^{2} \alpha=1 \\
& \quad \tan \beta=\tan \gamma \cdot \tan \alpha \\
& \quad \tan \theta_{x z}=\tan \theta_{y z} \cdot \tan \theta_{x y}
\end{aligned}
$$

2- Convex triangles:
3- Dynamic trigonometry:
4- Optical trigonometry :

$$
\left.\begin{array}{l}
V_{x}=\frac{x\left(x \cdot V_{x}+y \cdot V_{y}\right)}{x^{2}+y^{2}} \\
V_{Y}=\frac{y\left(x \cdot V_{x}+y \cdot V_{y}\right)}{x^{2}+y^{2}} \\
V_{z}=\frac{z\left(x \cdot V_{x}+z \cdot V_{z}\right)}{x^{2}+z^{2}}
\end{array}\right\} \Rightarrow V_{S}=\sqrt{V_{x}^{2}+V_{Y}^{2}+V_{3}^{2}}=300,00 \mathrm{~km} / \mathrm{sec}
$$



The above relationships have been substantiated throughout the chapter. If necessary, many problems can be calculated, such as the Euclidean trigonometry (Pythagoras), which is based on the relationship ( $\operatorname{Sin}^{2} \alpha+\operatorname{Cos}^{2} \alpha=1$ ) and has many applications in the science world.

