Types of Triangles

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Chapter I:

Point angles in space (a line that passes through the origin of coordinates)

(Mathematics)

Chapter II:

Linear angles in space (Mathematics)

Chapter III:

Angular velocity (Astronomy)

Introduction :

Metaphysics

1- I am not a member of any party, group or political group and I am not a politician at all. I am a member of the Association of Official Experts of Justice of Isfahan Province, and therefore, according to my bachelor's degree, I am committed to making my thoughts, words, and writings right. Therefore, one of my honors is that all scientific research has been done and its continuation is in the direction of truth and truth and in order to serve the world of humanity.

2- Real value: My real position, according to the proven scientific research, will be equivalent to the number one man at the end of the 21st century. This position is in terms of scientific value and is not comparable to the values of other disciplines.

3- Reasons for real value: It is stated at the beginning of my scientific site (www.p3m.ir).

Pythagoras + Newton + Einstein = Mahbod

$$A\begin{cases} Einstein: Proving\ the\ Fourth\ Dimension \\ Mahbod: Proving\ the\ fifth\ dimension \end{cases}$$

$$B\begin{cases} Newton: Proof\ of\ Differential\ Equations \\ Mahbod: Proof\ of\ differential\ equations\ by\ physics, mechanics\ and\ mathematics \end{cases}$$

$$C\begin{cases} Pythagoras: Proof\ of\ flat\ triangles \\ Mahbod: Proof\ of\ dynamic\ triangles\ and\ types\ of\ triangles \end{cases}$$

Proven types of triangles can now be used for greater accuracy, so that each point in space can be tested and controlled by the relationship of the types of triangles to several types of triangles and multiple angles. Due to the control of multiple angles, it can be used in ultra-modern computers to test and control electrons, etc., and finally quantum physics. By proving trigonometric relations in different types of trigonometry, we have solved some exercises and solved them clearly and simply, and it is worth mentioning that the basis of new mathematics in super-modern computers (types of trigonometry).

Chapter One :

Point angles in space (a line that passes through the origin of coordinates)

As the name of the book suggests, the main subject is the types of triangles, and we will examine them in the following. In this regard, we must first determine the types of point angles in space with respect to the coordinate axes, and then proceed to stabilize the relationships of the corresponding angles.

According to the coordinates of point (A) in space, the types of angles are as follows:

1. Point angle in space relative to 3 coordinate axes (ox, oy, oz)

1.1. The angle of the point relative to the coordinate axis (ox) is equal to (αx)

1.2. The angle of the point relative to the coordinate axis (oy) is equal to (αy)

1.3. The angle of the point relative to the coordinate axis (oz) is equal to (αz).

2. Point angle in space relative to 3 coordinate planes

2.1. The point angle relative to the coordinate plane (oxy) is equal to (αxy)

2.2. The angle of the point relative to the coordinate plane (oxz) is equal to (αxz).

2.3. The point angle relative to the coordinate plane (oyz) is equal to (αyz)

The point in space has 3 images in 3 axes of coordinates (oyz), (oxz), (oxy). Each of the points has an image on the coordinate plates and has an angle to the coordinate axes and can be written in order:

3. Point angle in space relative to 3 coordinate axes (ox, oy, oz)

3.1. The angle of the image point relative to the coordinate plane (oxy) is equal to (β)

3.2. The angle of the image point relative to the coordinate plane (oxz) is equal to (γ)

3.3. The angle of the image point relative to the coordinate plane (oyz) is equal to (θ)

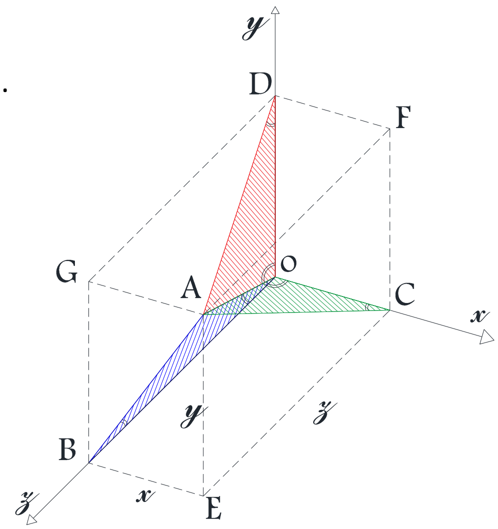
For clarity of angles and the power of the user's visualization, the corresponding angles in space relative to the 3 axes of the coordinates are shown in several forms, and for the point angles in space relative to the 3 pages, the coordinates are shown in several separate forms. (θ) and (γ) and (β) do not need to be represented.

To present and show the point in space, the images of the lines on the pages are presented as dotted lines (for easier visual embodiment power)

In the right-angled triangle $(\overset{\triangle}{\mathrm{AoC}})$. The angle $(\widehat{\mathrm{C}} = \frac{\pi}{2})$.

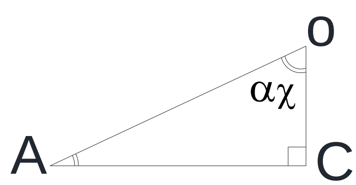
In the right triangle $(\overset{\triangle}{\mathrm{AoD}})$. The angle $\left(\widehat{\mathrm{D}} = \frac{\pi}{2}\right)$

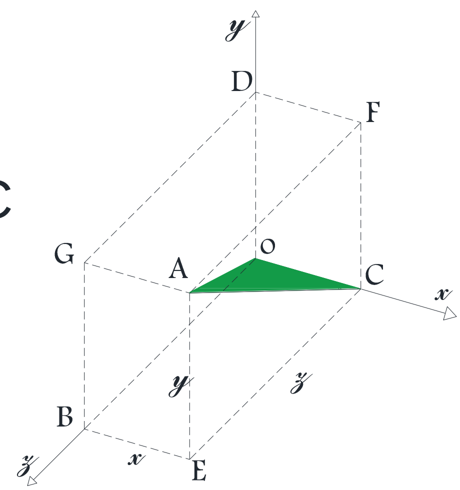
In the right triangle $(\overset{\triangle}{\mathrm{AoB}})$. The angle $(\widehat{\mathrm{B}} = \frac{\pi}{2})$



In the triangle $(\overset{\triangle}{\mathrm{AoC}})$

$$\widehat{\mathrm{A}} = (\frac{\pi}{2} - \alpha x)$$



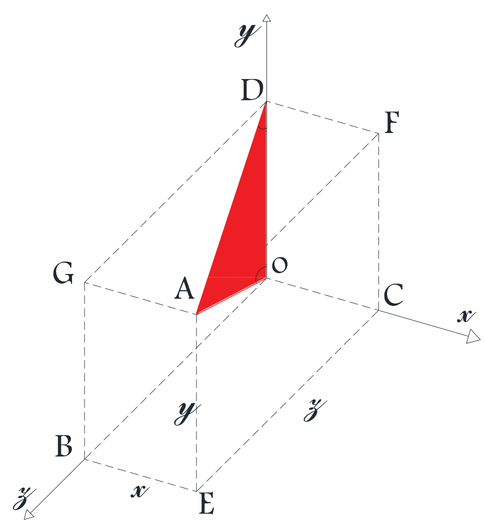


The origin of any point in space relative to the origin of the coordinates is equal to:

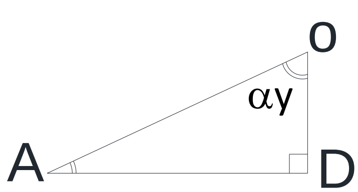
$$\begin{cases} \overline{\mathrm{oA}} = \sqrt{x^2 + y^2 + z^2} \\ \overline{oC} = x \end{cases}$$

$$\overline{\mathrm{oC}} = \overline{\mathrm{oA}}.\,Cos\ \alpha x$$

$$\text{ⓐ}\quad \boldsymbol{Cos\ \alpha x} = \frac{\overline{\boldsymbol{oC}}}{\overline{\mathbf{oA}}} = \frac{\boldsymbol{x}}{\sqrt{\boldsymbol{x^2+y^2+z^2}}}$$



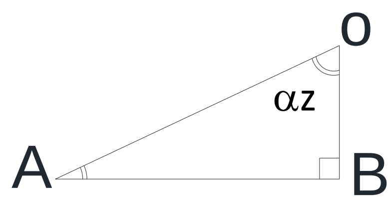
In the triangle $(\overset{\triangle}{\mathrm{AoD}})$



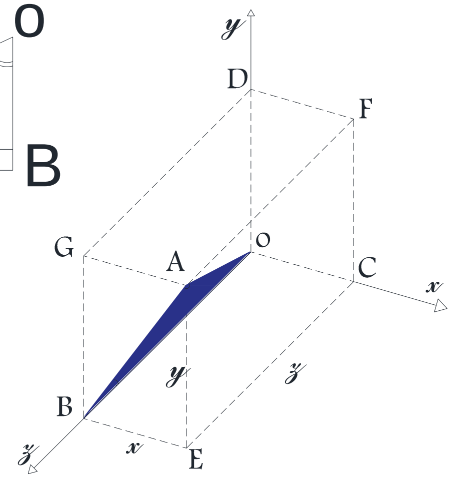
$$\begin{cases} \overline{\mathrm{oA}} = \sqrt{x^2 + y^2 + z^2} \\ \overline{oD} = y \end{cases}$$

$$\text{ⓑ}\quad \boldsymbol{Cos\ \alpha y} = \frac{\overline{\boldsymbol{oD}}}{\overline{\mathbf{oA}}} = \frac{\boldsymbol{y}}{\sqrt{\boldsymbol{x^2+y^2+z^2}}}$$

In the triangle ($\overset{\triangle}{AoB}$)



$$\begin{cases} \overline{oA} = \sqrt{x^2 + y^2 + z^2} \\ \overline{oB} = z \end{cases}$$



ⓒ $\boldsymbol{Cos\ \alpha y = \frac{\overline{oB}}{\overline{oA}} = \frac{z}{\sqrt{x^2+y^2+z^2}}}$

Now prove the relationship between ⓐ and ⓑ and ⓒ:

ⓐ $Cos^2\alpha x = \frac{x^2}{x^2+y^2+z^2}$

ⓑ $Cos^2\alpha y = \frac{y^2}{x^2+y^2+z^2}$

ⓒ $Cos^2\alpha z = \frac{z^2}{x^2+y^2+z^2}$

By summarizing the above relations, we can write:

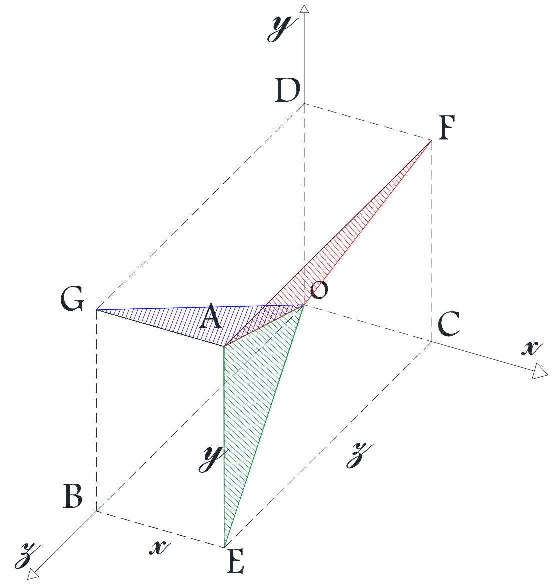
$$Cos^2\alpha x + Cos^2\alpha y + Cos^2\alpha z = \frac{x^2 + y^2 + z^2}{x^2 + y^2 + z^2} = 1$$

ⓓ $\boldsymbol{Cos^2\alpha x + Cos^2\alpha y + Cos^2\alpha z = 1}$

- In the right triangle $\overset{\triangle}{(AoF)}$. The angle$\left(\hat{F} = \frac{\pi}{2}\right)$

- In the right triangle $\overset{\triangle}{(AoE)}$. The angle$\left(\hat{E} = \frac{\pi}{2}\right)$

- In the right triangle $\overset{\triangle}{(AoG)}$. The angle$\left(\hat{G} = \frac{\pi}{2}\right)$

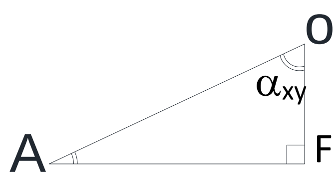


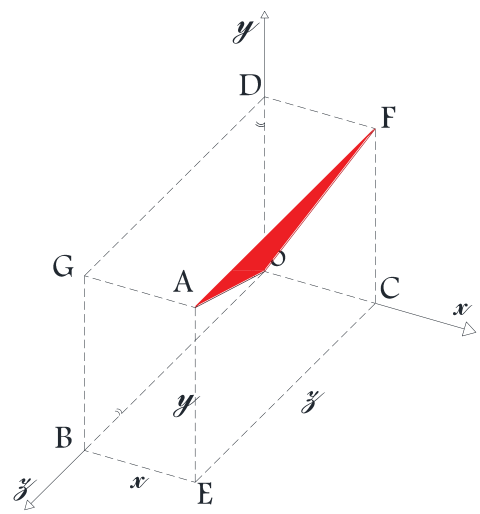
In the triangle $\overset{\triangle}{AoF}$:

$$\hat{F} = \frac{\pi}{2} \quad , \quad \hat{o} = \alpha_{xy}$$

$$\overline{oF} = \sqrt{x^2 + y^2}$$

$$\overline{oA} = \sqrt{x^2 + y^2 + z^2}$$



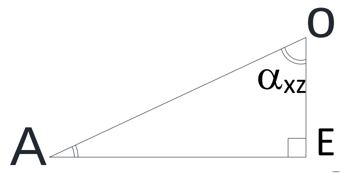


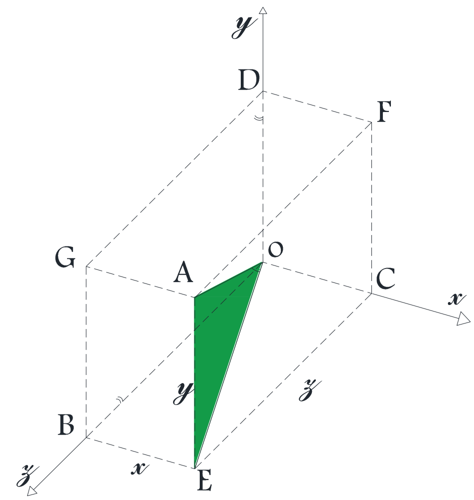
The distance of each point in space to the origin of the coordinates is equal to:

$$Cos\ \alpha_{xy} = \frac{\overline{oF}}{\overline{oA}} = \frac{\sqrt{x^2 + y^2}}{\sqrt{x^2 + y^2 + Z^2}}$$

ⓔ $\boldsymbol{Cos^2\ \alpha_{xy} = \frac{x^2+y^2}{x^2+y^2+Z^2}}$

In the triangle $\overset{\triangle}{AoE}$.:





$$\widehat{E} = \frac{\pi}{2} \quad , \quad \hat{o} = \alpha_{xz}$$

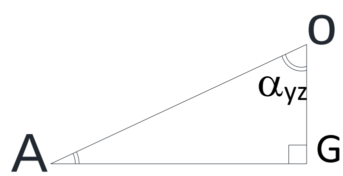
$$\overline{oE} = \sqrt{x^2 + z^2}$$

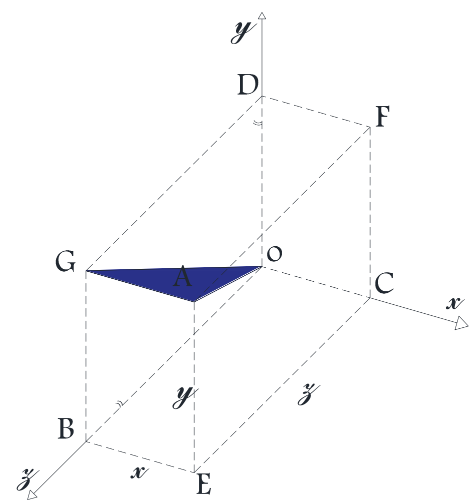
$$\overline{oA} = \sqrt{x^2 + y^2 + z^2}$$

$$Cos\ \alpha_{xz} = \frac{\overline{oE}}{\overline{oA}} = \frac{\sqrt{\boldsymbol{x^2+z^2}}}{\sqrt{\boldsymbol{x^2+y^2+Z^2}}}$$

$$ⓕ\ \boldsymbol{Cos^2\alpha_{xz}} = \frac{x^2+z^2}{x^2+y^2+Z^2}$$

In the triangle $\overset{\triangle}{AoG}$.:





$$\widehat{G} = \frac{\pi}{2} \quad , \quad \hat{o} = \alpha_{yz}$$

$$\overline{oG} = \sqrt{y^2 + z^2}$$

$$\overline{oA} = \sqrt{x^2 + y^2 + z^2}$$

$$Cos\ \alpha_{yz} = \frac{\overline{oG}}{\overline{oA}} = \frac{\sqrt{\boldsymbol{y^2+z^2}}}{\sqrt{\boldsymbol{x^2+y^2+Z^2}}}$$

$$ⓖ\ \boldsymbol{Cos^2\alpha_{yz}} = \frac{y^2+z^2}{x^2+y^2+Z^2}$$

Now, by adding above formulas (ⓔ + ⓕ + ⓖ = 1), we can conclude that the sum of the above three equations is equal to 1:

$$ⓗ\ \boldsymbol{Cos^{2\alpha_{xy}} + Cos^{2\alpha_{xz}} + Cos^{2\alpha_{yz}} = 1}$$

اینک ارتباط جزئیات فرمول ⓓ را با جزئیات فرمول به اثبات می رسانیم:

We now prove the relationship between the details of formula ⓓand the details of formula ⓗ:

$$ⓐ\ Cos\ \alpha x = \frac{x}{\sqrt{x^2+y^2+z^2}} \Rightarrow Cos^2\alpha x = \frac{x^2}{x^2+y^2+z^2}$$

$$ⓑ\ Cos\ \alpha y = \frac{y}{\sqrt{x^2+y^2+z^2}} \Rightarrow Cos^2\alpha y = \frac{y^2}{x^2+y^2+z^2}$$

$$ⓒ\ Cos\ \alpha z = \frac{z}{\sqrt{x^2+y^2+z^2}} \Rightarrow Cos^2\alpha z = \frac{z^2}{x^2+y^2+z^2}$$

$$Cos^2\alpha x + Cos^2\alpha y = \frac{x^2}{x^2+y^2+z^2} + \frac{y^2}{x^2+y^2+z^2} = \frac{x^2+y^2}{x^2+y^2+z^2} = Cos^2\alpha_{xy}$$

$$ⓘ\ \mathrm{Cos}^2\alpha_x + \mathrm{Cos}^2\alpha_y = \mathrm{Cos}^2\alpha_{xy}$$

$$Cos^2\alpha x + Cos^2\alpha z = \frac{x^2}{x^2+y^2+z^2} + \frac{z^2}{x^2+y^2+z^2} = \frac{x^2+z^2}{x^2+y^2+z^2} = \mathrm{Cos}^2\alpha_{xz}$$

$$ⓙ\ \mathrm{Cos}^2\alpha_x + \mathrm{Cos}^2\alpha_z = \mathrm{Cos}^2\alpha_{xz}$$

$$Cos^2\alpha y + Cos^2\alpha z = \frac{y^2}{x^2+y^2+z^2} + \frac{z^2}{x^2+y^2+z^2} = \frac{y^2+z^2}{x^2+y^2+z^2} = \mathrm{Cos}^2\alpha_{yz}$$

$$ⓚ\ \mathrm{Cos}^2\alpha_y + \mathrm{Cos}^2\alpha_z = \mathrm{Cos}^2\alpha_{yz}$$

Now to prove 3 angles θ , γ , β:

$$\left.\begin{array}{l} \frac{y}{x} = \tan\beta \\ \frac{z}{x} = \tan\gamma \\ \frac{z}{y} = \tan\theta \end{array}\right\} \Rightarrow \tan\gamma = \tan\theta \cdot \tan\beta \Rightarrow \frac{z}{x} = \frac{z}{y} \cdot \frac{y}{x}$$

$$\frac{z}{x}=\frac{\frac{z}{\sqrt{\boldsymbol{x^2+y^2+Z^2}}}}{\frac{x}{\sqrt{\boldsymbol{x^2+y^2+Z^2}}}}=\frac{Cos\ \alpha z}{Cos\ \alpha x}\Rightarrow\frac{z}{x}=\frac{Cos\ \alpha z}{Cos\ \alpha x}$$

$$\text{ⓛ}\ \boldsymbol{\frac{z}{x}=\tan\gamma=\frac{Cos\ \alpha z}{Cos\ \alpha x}}$$

$$\frac{z}{y}=\frac{\frac{z}{\sqrt{\boldsymbol{x^2+y^2+Z^2}}}}{\frac{y}{\sqrt{\boldsymbol{x^2+y^2+Z^2}}}}=\frac{Cos\ \alpha z}{Cos\ \alpha y}\Rightarrow\frac{z}{y}=\frac{Cos\ \alpha z}{Cos\ \alpha x}$$

$$\text{ⓛ}\ \boldsymbol{\frac{z}{x}=\tan\gamma=\frac{Cos\ \alpha z}{Cos\ \alpha x}}$$

$$\frac{z}{y}=\frac{\frac{z}{\sqrt{\boldsymbol{x^2+y^2+Z^2}}}}{\frac{y}{\sqrt{\boldsymbol{x^2+y^2+Z^2}}}}=\frac{Cos\ \alpha z}{Cos\ \alpha y}\Rightarrow\frac{z}{y}=\frac{Cos\ \alpha z}{Cos\ \alpha x}$$

$$\text{ⓜ}\ \boldsymbol{\frac{z}{x}=\tan\theta=\frac{Cos\ \alpha z}{Cos\ \alpha y}}$$

$$\frac{y}{x}=\frac{\frac{y}{\sqrt{\boldsymbol{x^2+y^2+Z^2}}}}{\frac{x}{\sqrt{\boldsymbol{x^2+y^2+Z^2}}}}=\frac{Cos\ \alpha y}{Cos\ \alpha x}\Rightarrow\frac{y}{x}=\frac{Cos\ \alpha y}{Cos\ \alpha x}$$

$$\text{ⓝ}\ \boldsymbol{\frac{y}{x}=\tan\beta=\frac{Cos\ \alpha y}{Cos\ \alpha x}}$$

Chapter II

Line angles in space

To examine the line in space, we need to examine how many types of lines we have:

Option One: Lines that extend beyond the origin of the coordinates.

Option 2: Lines that do not extend beyond the origin coordinates.

The first option:

In the first chapter, in order to prove the angles about the coordinates of point (A) in space, we performed the necessary studies and proved all the relationships between the angles. It is noteworthy that the coordinates of point (A) with respect to the origin of coordinates (o) have been proved. Therefore, in all types of triangles, the triangle is in the form of a right-angled triangle and the chord of the right-angled triangle is line $(\overline{oA})$. Therefore, all the angles of the line in space are the same as the angles of the point in space, and there is no need for re-stabilization. That is, the line $(\overline{oA})$ .Passes through the origin of the coordinates and $(\overline{oA})$. A line passes through the origin of the coordinates.

The second option

To prove the angles of a line in space, I need to have the coordinates of two points in space.

$$A\begin{cases}x_A\\y_A\\z_A\end{cases}\qquad B\begin{cases}x_B\\y_B\\z_B\end{cases}$$

Then, having the coordinates of two points in space, we can get the image of line $(\overline{\mathrm{AB}})$ in space in three pages of coordinates (oxy), (oxz), (oyz) with the image of the line, in 3 pages of coordinates we can equate Calculate the coordinates on each of the pages, and having the equation of each line in the three coordinates, get the line angle coefficient ⓜ($m\theta=\tan\theta$) , ⓛ ($m\gamma=\tan\gamma$) , ⓝ($m\beta=\tan\beta$)

Then you can get it by having formulas(ⓝ,ⓛ,ⓜ,ⓓ) Then you can get ($\cos\alpha x$,$\cos\alpha y$,$\cos\alpha z$) by having both values. (It is important to note that in order to facilitate the solution of the problem, it is necessary to place the origin of the coordinates in one of the points (A) or (B), and in this case, to solve the problem like the first chapter)

Then the values in the formulas ($\text{Cos}\alpha x$,$\text{Cos}\alpha y$,$\text{Cos}\alpha z$) can be calculated.

By solving a few examples, the solution will be more meaningful.

Practice 1:

$o\begin{vmatrix}0\\0\\0\end{vmatrix} \quad A\begin{vmatrix}7\\11\\13\end{vmatrix}$

Cos α_{xy}	$\sqrt{\frac{x^2+y^2}{x^2+y^2+z^2}}$	$=\sqrt{\frac{170}{339}}$
Cos² α_{xy}	$\frac{x^2+y^2}{x^2+y^2+z^2}$	$=\frac{170}{339}$
Sin² α_{xy}	1- Cos² α_{xy}	$=\frac{339}{339}-\frac{170}{339}=\frac{169}{339}$
Cos α_{xz}	$\sqrt{\frac{x^2+z^2}{x^2+y^2+z^2}}$	$=\sqrt{\frac{218}{339}}$
Cos² α_{xz}	$\frac{x^2+z^2}{x^2+y^2+z^2}$	$=\frac{218}{339}$
Sin² α_{xz}	1- Cos² α_{xz}	$=\frac{339}{339}-\frac{218}{339}=\frac{121}{339}$
Cos α_{yz}	$\sqrt{\frac{y^2+z^2}{x^2+y^2+z^2}}$	$=\sqrt{\frac{290}{339}}$
Cos² α_{yz}	$\frac{y^2+z^2}{x^2+y^2+z^2}$	$=\frac{290}{339}$
Sin² α_{yz}	1- Cos² α_{yz}	$=\frac{339}{339}-\frac{290}{339}=\frac{49}{339}$
Cos² α_{xy}+ Cos² α_{xz}+ Cos² α_{yz}=2		$=\frac{170}{339}+\frac{218}{339}+\frac{290}{339}=\frac{678}{339}=2$
Sin² α_{xy}+ Sin² α_{xz}+ Sin² α_{yz}=1		$=\frac{169}{339}+\frac{121}{339}+\frac{49}{339}=\frac{339}{339}=1$

Continue practicing 1:

$$o\left|\begin{matrix}0\\0\\0\end{matrix}\right. \qquad A\left|\begin{matrix}7\\11\\13\end{matrix}\right.$$

Cos α_{x}	$\frac{x}{\sqrt{x^2+y^2+z^2}}$	$=\frac{7}{\sqrt{49+121+169}}=\frac{7}{\sqrt{339}}$
Cos² α_{x}	$\frac{x^2}{x^2+y^2+z^2}$	$=\frac{49}{339}$
Sin² α_{x}	1- Cos² α_{xy}	$=\frac{290}{339}$
Cos α_{y}	$\frac{y}{\sqrt{x^2+y^2+z^2}}$	$=\frac{11}{\sqrt{339}}$
Cos² α_{y}	$\frac{y^2}{x^2+y^2+z^2}$	$=\frac{121}{339}$
Sin² α_{y}	1- Cos² α_{xz}	$=\frac{339-121}{339}=\frac{218}{339}$
Cos α_{z}	$\frac{z}{\sqrt{x^2+y^2+z^2}}$	$=\frac{13}{\sqrt{339}}$
Cos² α_{z}	$\frac{z^2}{x^2+y^2+z^2}$	$=\frac{169}{339}$
Sin² α_{z}	1- Cos² α_{yz}	$=\frac{339-169}{339}=\frac{170}{339}$ $=\frac{x^2+y^2+z^2}{x^2+y^2+z^2}-\frac{z^2}{x^2+y^2+z^2}$
Tan $\beta=\frac{y}{x}=\frac{Cos\ \alpha y}{Cos\ \alpha x}$		$=\frac{11}{7}$
Cos² α_{x}+ Cos² α_{y}+ Cos² α_{z}=1		$=\frac{49}{339}+\frac{121}{339}+\frac{169}{339}=\frac{339}{339}=1$

Practice 2:

$$o\left|\begin{matrix}0\\0\\0\end{matrix}\right. \qquad A\left|\begin{matrix}17\\19\\23\end{matrix}\right.$$

Cos α_{xy}	$\sqrt{\frac{x^2+y^2}{x^2+y^2+z^2}}$	$=\sqrt{\frac{289+361}{1179}}=\sqrt{\frac{650}{1179}}$
Cos² α_{xy}	$\frac{x^2+y^2}{x^2+y^2+z^2}$	$=\frac{650}{1179}$
Sin² α_{xy}	1- Cos² α_{xy}	$=\frac{1179}{1179}-\frac{650}{1179}=\frac{529}{1179}$
Cos α_{xz}	$\sqrt{\frac{x^2+z^2}{x^2+y^2+z^2}}$	$=\sqrt{\frac{289+529}{1179}}=\sqrt{\frac{818}{1179}}$
Cos² α_{xz}	$\frac{x^2+z^2}{x^2+y^2+z^2}$	$=\frac{818}{1179}$
Sin² α_{xz}	1- Cos² α_{xz}	$=\frac{1179}{1179}-\frac{818}{1179}=\frac{261}{1179}$
Cos α_{yz}	$\sqrt{\frac{y^2+z^2}{x^2+y^2+z^2}}$	$=\sqrt{\frac{361+529}{1179}}=\sqrt{\frac{890}{1179}}$
Cos² α_{yz}	$\frac{y^2+z^2}{x^2+y^2+z^2}$	$=\frac{890}{1179}$
Sin² α_{yz}	1- Cos² α_{yz}	$=\frac{1179}{1179}-\frac{890}{1179}=\frac{289}{1179}$
Cos² α_{xy}+ Cos² α_{xz}+ Cos² α_{yz}=2		$=\frac{650}{1179}+\frac{818}{1179}+\frac{890}{1179}=\frac{2358}{1179}=2$
Sin² α_{xy}+ Sin² α_{xz}+ Sin² α_{yz}=1		$=\frac{529}{1179}+\frac{361}{1179}+\frac{289}{1179}=\frac{1179}{1179}=1$

Continue practicing 2:

$o\left|\begin{matrix}0\\0\\0\end{matrix}\right.\qquad A\left|\begin{matrix}17\\19\\23\end{matrix}\right.$

Cos α_x	$\frac{x}{\sqrt{x^2+y^2+z^2}}$	$=\frac{17}{\sqrt{289+361+529}}=\frac{17}{\sqrt{1179}}$
Cos² α_x	$\frac{x^2}{x^2+y^2+z^2}$	$=\frac{289}{1179}$
Sin² α_x	1- Cos² α_{xy}	$=\frac{1179}{1179}-\frac{289}{1179}=\frac{890}{1179}$
Cos α_y	$\frac{y}{\sqrt{x^2+y^2+z^2}}$	$=\frac{19}{\sqrt{1179}}$
Cos² α_y	$\frac{y^2}{x^2+y^2+z^2}$	$=\frac{361}{1179}$
Sin² α_y	1- Cos² α_{xz}	$=\frac{1179}{1179}-\frac{361}{1179}=\frac{818}{1179}$
Cos α_z	$\frac{z}{\sqrt{x^2+y^2+z^2}}$	$=\frac{23}{\sqrt{1179}}$
Cos² α_z	$\frac{z^2}{x^2+y^2+z^2}$	$=\frac{529}{1179}$
Sin² α_z	1- Cos² α_{xz}	$=\frac{1179}{1179}=\frac{529}{1179}=\frac{650}{1179}$
Tan $\beta=\frac{y}{x}=\frac{Cos\ \alpha y}{Cos\ \alpha x}$		$=\frac{\frac{19}{\sqrt{1179}}}{\frac{17}{\sqrt{1179}}}=\frac{19}{17}$
Cos² α_x+ Cos² α_y+ Cos² α_z=1		$=\frac{289}{1179}+\frac{261}{1179}+\frac{529}{1179}=\frac{1179}{1179}=1$

Practice 3:

$$o\left|\begin{matrix}0\\0\\0\end{matrix}\right. \qquad A\left|\begin{matrix}29\\31\\37\end{matrix}\right.$$

Cos α_{xy}	$\sqrt{\dfrac{x^2+y^2}{x^2+y^2+z^2}}$	$=\sqrt{\frac{841+961}{3171}}=\sqrt{\frac{1802}{1179}}$
Cos² α_{xy}	$\dfrac{x^2+y^2}{x^2+y^2+z^2}$	$=\dfrac{1802}{3171}$
Sin² α_{xy}	1- Cos² α_{xy}	$=\dfrac{3171}{3171}-\dfrac{1802}{3171}=\dfrac{1369}{3171}$
Cos α_{xz}	$\sqrt{\dfrac{x^2+z^2}{x^2+y^2+z^2}}$	$=\sqrt{\frac{841+1369}{3171}}=\sqrt{\frac{2210}{3171}}$
Cos² α_{xz}	$\dfrac{x^2+z^2}{x^2+y^2+z^2}$	$=\dfrac{2210}{3171}$
Sin² α_{xz}	1- Cos² α_{xz}	$=\dfrac{3171}{3171}-\dfrac{2210}{3171}=\dfrac{961}{3171}$
Cos α_{yz}	$\sqrt{\dfrac{y^2+z^2}{x^2+y^2+z^2}}$	$=\sqrt{\frac{961+1369}{3171}}=\sqrt{\frac{2330}{3171}}$
Cos² α_{yz}	$\dfrac{y^2+z^2}{x^2+y^2+z^2}$	$=\dfrac{2330}{3171}$
Sin² α_{yz}	1- Cos² α_{yz}	$=\dfrac{3171}{3171}-\dfrac{2330}{3171}=\dfrac{841}{3171}$
Cos² α_{xy}+ Cos² α_{xz}+ Cos² α_{yz}=2		$=\frac{1802}{3171}+\frac{2210}{3171}+\frac{2330}{3171}=\frac{6340}{3171}=2$
Sin² α_{xy}+ Sin² α_{xz}+ Sin² α_{yz}=1		$=\frac{1369}{3171}+\frac{961}{3171}+\frac{841}{3171}=\frac{3171}{3171}=1$

Continue practicing 3:

$o\begin{vmatrix}0\\0\\0\end{vmatrix} \quad A\begin{vmatrix}29\\31\\37\end{vmatrix}$

Cos α_x	$\frac{x}{\sqrt{x^2+y^2+z^2}}$	$=\frac{29}{\sqrt{289+361+529}}=\frac{29}{\sqrt{3171}}$
Cos² α_x	$\frac{x^2}{x^2+y^2+z^2}$	$=\frac{841}{3171}$
Sin² α_x	1- Cos² α_{xy}	$=\frac{3171}{3171}-\frac{841}{3171}=\frac{2330}{3171}$
Cos α_y	$\frac{y}{\sqrt{x^2+y^2+z^2}}$	$=\frac{31}{\sqrt{3171}}$
Cos² α_y	$\frac{y^2}{x^2+y^2+z^2}$	$=\frac{961}{3171}$
Sin² α_y	1- Cos² α_{xz}	$=\frac{3171}{3171}-\frac{960}{3171}=\frac{2210}{3171}$
Cos α_z	$\frac{z}{\sqrt{x^2+y^2+z^2}}$	$=\frac{37}{\sqrt{3171}}$
Cos² α_z	$\frac{z^2}{x^2+y^2+z^2}$	$=\frac{1369}{3171}$
Sin² α_z	1- Cos² α_{xz}	$=\frac{3171}{3171}=\frac{1369}{3171}=\frac{1802}{3171}$
Tan $\beta=\frac{y}{x}=\frac{Cos\ \alpha y}{Cos\ \alpha x}$		$=\frac{31}{29}=\frac{\frac{31}{\sqrt{3171}}}{\frac{29}{\sqrt{3171}}}=\frac{31}{29}$
Cos² α_x+ Cos² α_y+ Cos² α_z=1		$=\frac{841}{3171}+\frac{961}{3171}+\frac{1369}{3171}=\frac{3171}{3171}=1$

Practice 4:

$o\left|\begin{matrix}0\\0\\0\end{matrix}\right.\quad A\left|\begin{matrix}41\\43\\47\end{matrix}\right.$

Cos α_{xy}	$\sqrt{\dfrac{x^2+y^2}{x^2+y^2+z^2}}$	$=\sqrt{\frac{1681+1849}{5739}}=\sqrt{\frac{3530}{5739}}$
Cos² α_{xy}	$\dfrac{x^2+y^2}{x^2+y^2+z^2}$	$=\dfrac{3530}{5739}$
Sin² α_{xy}	1- Cos² α_{xy}	$=\dfrac{5739}{5739}-\dfrac{3530}{5739}=\dfrac{2209}{5739}$
Cos α_{xz}	$\sqrt{\dfrac{x^2+z^2}{x^2+y^2+z^2}}$	$=\sqrt{\frac{1681+2209}{5739}}=\sqrt{\frac{3890}{5739}}$
Cos² α_{xz}	$\dfrac{x^2+z^2}{x^2+y^2+z^2}$	$=\dfrac{3890}{5739}$
Sin² α_{xz}	1- Cos² α_{xz}	$=\dfrac{5739}{5739}-\dfrac{3890}{5739}=\dfrac{1849}{5739}$
Cos α_{yz}	$\sqrt{\dfrac{y^2+z^2}{x^2+y^2+z^2}}$	$=\sqrt{\frac{1849+2209}{5739}}=\sqrt{\frac{4058}{5739}}$
Cos² α_{yz}	$\dfrac{y^2+z^2}{x^2+y^2+z^2}$	$=\dfrac{4058}{5739}$
Sin² α_{yz}	1- Cos² α_{yz}	$=\dfrac{5739}{5739}-\dfrac{4058}{5739}=\dfrac{1681}{5739}$
Cos² α_{xy}+ Cos² α_{xz}+ Cos² α_{yz}=2		$=\frac{3530}{5739}+\frac{3890}{5739}+\frac{4058}{5739}=\frac{11478}{5739}=2$
Sin² α_{xy}+ Sin² α_{xz}+ Sin² α_{yz}=1		$=\frac{2209}{5739}+\frac{1849}{5739}+\frac{1681}{5739}=\frac{5739}{5739}=1$

Continue practicing 4:

$$o\left|\begin{matrix}0\\0\\0\end{matrix}\right. \qquad A\left|\begin{matrix}41\\43\\47\end{matrix}\right.$$

Cos α_x	$\frac{x}{\sqrt{x^2+y^2+z^2}}$	$=\frac{41}{\sqrt{1681+1849+2209}}=\frac{41}{\sqrt{5739}}$
Cos² α_x	$\frac{x^2}{x^2+y^2+z^2}$	$=\frac{1681}{5739}$
Sin² α_x	1- Cos² α_{xy}	$=\frac{5739}{5739}-\frac{1681}{5739}=\frac{4058}{5739}$
Cos α_y	$\frac{y}{\sqrt{x^2+y^2+z^2}}$	$=\frac{43}{\sqrt{5739}}$
Cos² α_y	$\frac{y^2}{x^2+y^2+z^2}$	$=\frac{1849}{5739}$
Sin² α_y	1- Cos² α_{xz}	$=\frac{5739}{5739}-\frac{1849}{5739}=\frac{3890}{5739}$
Cos α_z	$\frac{z}{\sqrt{x^2+y^2+z^2}}$	$=\frac{47}{\sqrt{5739}}$
Cos² α_z	$\frac{z^2}{x^2+y^2+z^2}$	$=\frac{2209}{5739}$
Sin² α_z	1- Cos² α_{xz}	$=\frac{5739}{5739}=\frac{2209}{5739}=\frac{3530}{5739}$
Tan β=$\frac{y}{x}=\frac{Cos\ \alpha y}{Cos\ \alpha x}$		$=\frac{43}{41}=\frac{\frac{43}{\sqrt{5739}}}{\frac{41}{\sqrt{5739}}}=\frac{43}{41}$
Cos² α_x+ Cos² α_y+ Cos² α_z=1		$=\frac{1681}{5739}+\frac{1849}{5739}+\frac{2209}{5739}=\frac{5729}{5739}=1$

Practice 5:

$$o\left|\begin{matrix}0\\0\\0\end{matrix}\right. \qquad A\left|\begin{matrix}49\\51\\53\end{matrix}\right.$$

Cos α_{xy}	$\sqrt{\dfrac{x^2+y^2}{x^2+y^2+z^2}}$	$=\sqrt{\frac{2401+2601}{7811}}=\sqrt{\frac{5002}{7811}}$
Cos² α_{xy}	$\dfrac{x^2+y^2}{x^2+y^2+z^2}$	$=\dfrac{5002}{7811}$
Sin² α_{xy}	1- Cos² α_{xy}	$=\dfrac{7811}{7811}-\dfrac{5002}{7811}=\dfrac{2809}{7811}$
Cos α_{xz}	$\sqrt{\dfrac{x^2+z^2}{x^2+y^2+z^2}}$	$=\sqrt{\frac{2401+2809}{7811}}=\sqrt{\frac{5210}{7811}}$
Cos² α_{xz}	$\dfrac{x^2+z^2}{x^2+y^2+z^2}$	$=\dfrac{5210}{7811}$
Sin² α_{xz}	1- Cos² α_{xz}	$=\dfrac{7811}{7811}-\dfrac{5210}{7811}=\dfrac{2601}{7811}$
Cos α_{yz}	$\sqrt{\dfrac{y^2+z^2}{x^2+y^2+z^2}}$	$=\sqrt{\frac{2601+2809}{7811}}=\sqrt{\frac{5410}{7811}}$
Cos² α_{yz}	$\dfrac{y^2+z^2}{x^2+y^2+z^2}$	$=\dfrac{5410}{7811}$
Sin² α_{yz}	1- Cos² α_{yz}	$=\dfrac{7811}{7811}-\dfrac{5410}{7811}=\dfrac{2401}{7811}$
Cos² α_{xy}+ Cos² α_{xz}+ Cos² α_{yz}=2		$=\frac{5002}{7811}+\frac{5210}{7811}+\frac{5410}{7811}=\frac{15622}{7811}=2$
Sin² α_{xy}+ Sin² α_{xz}+ Sin² α_{yz}=1		$=\frac{2809}{7811}+\frac{2601}{7811}+\frac{2401}{7811}=\frac{7811}{7811}=1$

Continue practicing 5:

$o\left|\begin{matrix}0\\0\\0\end{matrix}\right. \qquad A\left|\begin{matrix}49\\51\\53\end{matrix}\right.$

Cos α_x	$\frac{x}{\sqrt{x^2+y^2+z^2}}$	$=\frac{49}{\sqrt{2401+2601+2809}}=\frac{49}{\sqrt{7811}}$
Cos² α_x	$\frac{x^2}{x^2+y^2+z^2}$	$=\frac{2401}{7811}$
Sin² α_x	1- Cos² α_{xy}	$=\frac{7811}{7811}-\frac{2401}{7811}=\frac{5401}{7811}$
Cos α_y	$\frac{y}{\sqrt{x^2+y^2+z^2}}$	$=\frac{51}{\sqrt{7811}}$
Cos² α_y	$\frac{y^2}{x^2+y^2+z^2}$	$=\frac{2601}{7811}$
Sin² α_y	1- Cos² α_{xz}	$=\frac{7811}{7811}-\frac{2601}{7811}=\frac{5211}{7811}$
Cos α_z	$\frac{z}{\sqrt{x^2+y^2+z^2}}$	$=\frac{53}{\sqrt{7811}}$
Cos² α_z	$\frac{z^2}{x^2+y^2+z^2}$	$=\frac{2809}{7811}$
Sin² α_z	1- Cos² α_{xz}	$=\frac{7811}{7811}=\frac{2809}{7811}=\frac{5002}{7811}$
Tan β=$\frac{y}{x}=\frac{Cos\ \alpha y}{Cos\ \alpha x}$		$=\frac{51}{49}=\frac{\frac{51}{\sqrt{7811}}}{\frac{49}{\sqrt{7811}}}=\frac{51}{49}$
Cos² α_x+ Cos² α_y+ Cos² α_z=1		$=\frac{2401}{7811}+\frac{2601}{7811}+\frac{2809}{7811}=\frac{7811}{7811}=1$

Practice 6:

$$o\begin{vmatrix}0\\0\\0\end{vmatrix} \quad A\begin{vmatrix}57\\59\\61\end{vmatrix}$$

Cos α_{xy}	$\sqrt{\frac{x^2+y^2}{x^2+y^2+z^2}}$	$=\sqrt{\frac{3249+3481}{10451}}=\sqrt{\frac{6730}{10451}}$
Cos² α_{xy}	$\frac{x^2+y^2}{x^2+y^2+z^2}$	$=\frac{6730}{10451}$
Sin² α_{xy}	1- Cos² α_{xy}	$=\frac{10451}{10451}-\frac{6730}{10451}=\frac{3721}{10451}$
Cos α_{xz}	$\sqrt{\frac{x^2+z^2}{x^2+y^2+z^2}}$	$=\sqrt{\frac{3249+3721}{10451}}=\sqrt{\frac{6970}{10451}}$
Cos² α_{xz}	$\frac{x^2+z^2}{x^2+y^2+z^2}$	$=\frac{6970}{10451}$
Sin² α_{xz}	1- Cos² α_{xz}	$=\frac{10451}{10451}-\frac{6970}{10451}=\frac{3481}{10451}$
Cos α_{yz}	$\sqrt{\frac{y^2+z^2}{x^2+y^2+z^2}}$	$=\sqrt{\frac{3481+3371}{10451}}=\sqrt{\frac{7202}{10451}}$
Cos² α_{yz}	$\frac{y^2+z^2}{x^2+y^2+z^2}$	$=\frac{7202}{10451}$
Sin² α_{yz}	1- Cos² α_{yz}	$=\frac{10451}{10451}-\frac{7202}{10451}=\frac{3249}{10451}$
Cos² α_{xy}+ Cos² α_{xz}+ Cos² α_{yz}=2		$=\frac{6730}{10451}+\frac{6970}{10451}+\frac{7202}{10451}=\frac{20902}{10451}=2$
Sin² α_{xy}+ Sin² α_{xz}+ Sin² α_{yz}=1		$=\frac{3721}{10451}+\frac{3481}{10451}+\frac{3249}{10451}=\frac{10451}{10451}=1$

Continue practicing 6:

$o\left|\begin{matrix}0\\0\\0\end{matrix}\right.\quad A\left|\begin{matrix}57\\59\\61\end{matrix}\right.$

Cos α_x	$\frac{x}{\sqrt{x^2+y^2+z^2}}$	$=\frac{57}{\sqrt{2401+2601+2809}}=\frac{57}{\sqrt{10451}}$
Cos² α_x	$\frac{x^2}{x^2+y^2+z^2}$	$=\frac{3249}{10451}$
Sin² α_x	1- Cos² α_{xy}	$=\frac{10451}{10451}-\frac{3249}{10451}=\frac{7202}{10451}$
Cos α_y	$\frac{y}{\sqrt{x^2+y^2+z^2}}$	$=\frac{59}{\sqrt{10451}}$
Cos² α_y	$\frac{y^2}{x^2+y^2+z^2}$	$=\frac{3481}{10451}$
Sin² α_y	1- Cos² α_{xz}	$=\frac{10451}{10451}-\frac{3481}{10451}=\frac{6970}{10451}$
Cos α_z	$\frac{z}{\sqrt{x^2+y^2+z^2}}$	$=\frac{61}{\sqrt{10451}}$
Cos² α_z	$\frac{z^2}{x^2+y^2+z^2}$	$=\frac{3721}{10451}$
Sin² α_z	1- Cos² α_{xz}	$=\frac{10451}{10451}=\frac{3721}{10451}=\frac{6730}{10451}$
Tan β=$\frac{y}{x}=\frac{Cos\ \alpha y}{Cos\ \alpha x}$		$=\frac{59}{57}=\frac{\frac{59}{\sqrt{10451}}}{\frac{57}{\sqrt{10451}}}=\frac{59}{57}$
Cos² α_x+ Cos² α_y+ Cos² α_z=1		$=\frac{3249}{10451}+\frac{3481}{10451}+\frac{3721}{10451}=\frac{10451}{10451}=1$

Practice 7:

X=4
Y=8
Z=16

Cos α_X	$\frac{X}{\sqrt{X^2+Y^2+Z^2}}$	$=\frac{4}{\sqrt{16+64+256}}=\frac{4}{\sqrt{336}}$
$\text{Cos}^2\ \alpha_X$	$\frac{X^2}{X^2+Y^2+Z^2}$	$=\frac{16}{336}$
$\text{Sin}^2\ \alpha_X$	1- $\text{Cos}^2\ \alpha_X$	$=\frac{336}{336}-\frac{16}{336}=\frac{320}{336}$
Cos α_Y	$\frac{Y}{\sqrt{X^2+Y^2+Z^2}}$	$=\frac{8}{\sqrt{336}}$
$\text{Cos}^2\ \alpha_Y$	$\frac{Y^2}{X^2+Y^2+Z^2}$	$=\frac{64}{336}$
$\text{Sin}^2\ \alpha_Y$	1- $\text{Cos}^2\ \alpha_Y$	$=\frac{336}{336}-\frac{64}{336}=\frac{272}{336}$
Cos α_Z	$\frac{Z}{\sqrt{X^2+Y^2+Z^2}}$	$=\frac{16}{\sqrt{336}}$
$\text{Cos}^2\ \alpha_Z$	$\frac{Z^2}{X^2+Y^2+Z^2}$	$=\frac{259}{336}$
$\text{Sin}^2\ \alpha_Z$	1- $\text{Cos}^2\ \alpha_Z$	$=\frac{336}{336}=\frac{256}{336}=\frac{80}{336}$
tan $\beta=\frac{Y}{X}=\frac{\text{Cos}\ \alpha_Y}{\text{Cos}\ \alpha_X}$		$=\frac{8}{4}=\frac{\frac{8}{\sqrt{336}}}{\frac{4}{\sqrt{336}}}=\frac{8}{4}=2$
$\text{Cos}^2\ \alpha_X+\text{Cos}^2\ \alpha_Y+\text{Cos}^2\ \alpha_Z=1$		$=\frac{16}{336}+\frac{64}{336}+\frac{256}{336}=\frac{336}{336}=1$

Continue practicing 7:

$$A\left|\begin{matrix}13\\11\\7\end{matrix}\right. \qquad B\left|\begin{matrix}17\\19\\23\end{matrix}\right. \qquad \begin{matrix}\alpha = 13\\ \beta = 11\\ \gamma = 7\end{matrix} \quad \begin{matrix}x = \mathrm{X} + \alpha \Rightarrow \mathrm{X} = x - \alpha = 17 - 13 = 4\\ y = \mathrm{Y} + \beta \Rightarrow \mathrm{Y} = \mathrm{y} - \beta = 19 - 11 = 8\\ z = \mathrm{Z} + \gamma \Rightarrow \mathrm{Z} = \mathrm{z} - \gamma = 23 - 7 = 16\end{matrix}$$

$$\begin{matrix}x = 4\\ y = 8\\ z = 16\end{matrix} \qquad o\left|\begin{matrix}0\\0\\0\end{matrix}\right.$$

Cos α_{xy}	$\sqrt{\frac{X^2+Y^2}{X^2+Y^2+Z^2}}$	$=\frac{\sqrt{80}}{\sqrt{16+64+256}}=\frac{\sqrt{80}}{\sqrt{336}}$
Cos² α_{xy}	$\frac{X^2+Y^2}{X^2+Y^2+Z^2}$	$=\frac{80}{336}$
Sin² α_{xy}	1- COS² α_{XY}	$=\frac{336}{336}-\frac{80}{336}=\frac{256}{336}$
Cos α_{xz}	$\sqrt{\frac{X^2+Z^2}{X^2+Y^2+Z^2}}$	$=\sqrt{\frac{16+256}{336}}=\sqrt{\frac{272}{336}}$
Cos² α_{xz}	$\frac{X^2+Z^2}{X^2+Y^2+Z^2}$	$=\frac{272}{336}$
Sin² α_{xz}	1- COS² α_{XZ}	$=\frac{336}{336}-\frac{272}{336}=\frac{64}{336}$
Cos α_{yz}	$\sqrt{\frac{Y^2+Z^2}{X^2+Y^2+Z^2}}$	$=\sqrt{\frac{64+256}{336}}=\sqrt{\frac{320}{336}}$
Cos² α_{yz}	$\frac{Y^2+Z^2}{X^2+Y^2+Z^2}$	$=\frac{7202}{336}$
Sin² α_{yz}	1- COS² α_{YZ}	$=\frac{336}{336}-\frac{320}{336}=\frac{16}{336}$
Cos² α_{xy}+ Cos² α_{xz}+ Cos² α_{yz}=2		$=\frac{80}{336}+\frac{272}{336}+\frac{320}{336}=\frac{672}{10336}=2$
Sin² α_{xy}+ Sin² α_{xz}+ Sin² α_{yz}=1		$=\frac{256}{336}+\frac{64}{336}+\frac{16}{336}=\frac{336}{336}=1$
tan α_{XY}=$\frac{Z}{\sqrt{X^2+Y^2}}$ TAN α_{XZ}=$\frac{Y}{\sqrt{X^2+Z^2}}$ TAN α_{YZ}=$\frac{X}{\sqrt{Y^2+Z^2}}$		

Practice 8:

$$A\left|\begin{matrix}17\\19\\23\end{matrix}\right. \qquad B\left|\begin{matrix}107\\111\\113\end{matrix}\right.$$

$$x = \mathrm{X} + \alpha \Rightarrow \mathrm{X} = x + \alpha = 17 \qquad \mathrm{X} = 107 - 17 = 90 \Rightarrow X = 90$$

$$y = \mathrm{Y} + \beta \Rightarrow \mathrm{Y} = \mathrm{y} + \beta = 19 \qquad \mathrm{Y} = 111 - 19 = 92 \Rightarrow \mathrm{Y} = 92$$

$$z = \mathrm{Z} + \gamma \Rightarrow \mathrm{Z} = \mathrm{z} + \gamma = 23 \qquad \mathrm{Z} = 113 - 23 = 90 \Rightarrow \mathrm{Z} = 90$$

Cos α_X	$\frac{\mathrm{X}}{\sqrt{\mathrm{X}^2+\mathrm{Y}^2+\mathrm{Z}^2}}$	$=\frac{90}{\sqrt{8100+8464+8100}} = \frac{90}{\sqrt{24664}}$
Cos² α_X	$\frac{\mathrm{X}^2}{\mathrm{X}^2+\mathrm{Y}^2+\mathrm{Z}^2}$	$=\frac{8100}{24664}$
Sin² α_X	1- Cos² α_X	$=\frac{24664}{24664}-\frac{8100}{24664}=\frac{16564}{24664}$
Cos α_Y	$\frac{\mathrm{Y}}{\sqrt{\mathrm{X}^2+\mathrm{Y}^2+\mathrm{Z}^2}}$	$=\frac{92}{\sqrt{24664}}$
Cos² α_Y	$\frac{\mathrm{Y}^2}{\mathrm{X}^2+\mathrm{Y}^2+\mathrm{Z}^2}$	$=\frac{8464}{24664}$
Sin² α_Y	1- Cos² α_Y	$=\frac{24664}{24664}-\frac{8464}{24664}=\frac{16200}{24664}$
Cos α_Z	$\frac{\mathrm{Z}}{\sqrt{\mathrm{X}^2+\mathrm{Y}^2+\mathrm{Z}^2}}$	$=\frac{90}{\sqrt{24664}}$
Cos² α_Z	$\frac{\mathrm{Z}^2}{\mathrm{X}^2+\mathrm{Y}^2+\mathrm{Z}^2}$	$=\frac{8100}{24664}$
Sin² α_Z	1- Cos² α_Z	$=\frac{24664}{24664}=\frac{8100}{24664}=\frac{16564}{24664}$
tan $\beta=\frac{\mathrm{Y}}{\mathrm{X}}=\frac{\mathrm{Cos}\ \alpha_{\mathrm{Y}}}{\mathrm{Cos}\ \alpha_{\mathrm{X}}}$		$=\frac{92}{90}=\frac{\frac{92}{\sqrt{24664}}}{\frac{90}{\sqrt{24664}}}=\frac{92}{90}$
Cos² α_X+ Cos² α_Y+ Cos² α_Z=1		$=\frac{8100}{24664}+\frac{8464}{24664}+\frac{8100}{24664}=\frac{24664}{24664}=1$

Continue practicing 8:

$A\begin{vmatrix}17\\19\\23\end{vmatrix} \quad B\begin{vmatrix}107\\111\\113\end{vmatrix}$

$X = 90$ $\quad o\begin{vmatrix}0\\0\\0\end{vmatrix}$
$Y = 92$
$Z = 90$

Cos α_{xy}	$\sqrt{\dfrac{X^2+Y^2}{X^2+Y^2+Z^2}}$	$=\dfrac{\sqrt{8100+8464}}{\sqrt{24664}}=\dfrac{\sqrt{16564}}{\sqrt{24664}}$
Cos² α_{xy}	$\dfrac{X^2+Y^2}{X^2+Y^2+Z^2}$	$=\dfrac{16564}{24664}$
Sin² α_{xy}	1- COS² α_{XY}	$=\dfrac{24664}{24664}-\dfrac{16564}{24664}=\dfrac{8100}{24664}$
Cos α_{xz}	$\sqrt{\dfrac{X^2+Z^2}{X^2+Y^2+Z^2}}$	$=\sqrt{\dfrac{16200}{24664}}$
Cos² α_{xz}	$\dfrac{X^2+Z^2}{X^2+Y^2+Z^2}$	$=\dfrac{16200}{24664}$
Sin² α_{xz}	1- COS² α_{XZ}	$=\dfrac{24664}{24664}-\dfrac{16200}{24664}=\dfrac{8464}{24664}$
Cos α_{yz}	$\sqrt{\dfrac{Y^2+Z^2}{X^2+Y^2+Z^2}}$	$=\sqrt{\dfrac{8464+8100}{24664}}=\sqrt{\dfrac{16564}{24664}}$
Cos² α_{yz}	$\dfrac{Y^2+Z^2}{X^2+Y^2+Z^2}$	$=\dfrac{16564}{24664}$
Sin² α_{yz}	1- COS² α_{YZ}	$=\dfrac{24664}{24664}-\dfrac{16564}{24664}=\dfrac{8100}{24664}$
Cos² α_{xy}+ Cos² α_{xz}+ Cos² α_{yz}=2		$=\dfrac{16564}{24664}+\dfrac{16200}{24664}+\dfrac{16564}{24664}=\dfrac{49328}{24664}=2$
Sin² α_{xy}+ Sin² α_{xz}+ Sin² α_{yz}=1		$=\dfrac{8100}{24664}+\dfrac{8464}{24664}+\dfrac{8100}{24664}=\dfrac{24664}{24664}=1$

tan $\alpha_{xy}=\dfrac{Z}{\sqrt{x^2+y^2}}=\dfrac{90}{\sqrt{16564}}$ tan $\alpha_{xz}=\dfrac{Y}{\sqrt{x^2+z^2}}=\dfrac{92}{\sqrt{16200}}$ tan $\alpha_{YZ}=\dfrac{X}{\sqrt{y^2+Z^2}}=\dfrac{90}{\sqrt{16564}}$

Practice 9:

$$A\left|\begin{matrix}29\\31\\37\end{matrix}\right. \qquad B\left|\begin{matrix}97\\101\\103\end{matrix}\right. \qquad \begin{matrix}x=X+29\\y=Y+31\\z=Z+37\end{matrix} \qquad \begin{matrix}\mathrm{X}=97-29=68\\\mathrm{Y}=101-31=71\\\mathrm{Z}=103-37=66\end{matrix}$$

Cos α_X	$\frac{\mathrm{X}}{\sqrt{\mathrm{X}^2+\mathrm{Y}^2+\mathrm{Z}^2}}$	$=\frac{68}{\sqrt{4624+5041+4356}}=\frac{68}{\sqrt{14021}}$
Cos² α_X	$\frac{\mathrm{X}^2}{\mathrm{X}^2+\mathrm{Y}^2+\mathrm{Z}^2}$	$=\frac{4624}{14021}$
Sin² α_X	1- Cos² α_X	$=\frac{14021}{14021}-\frac{4624}{14021}=\frac{9577}{14021}$
Cos α_Y	$\frac{\mathrm{Y}}{\sqrt{\mathrm{X}^2+\mathrm{Y}^2+\mathrm{Z}^2}}$	$=\frac{71}{\sqrt{14021}}$
Cos² α_Y	$\frac{\mathrm{Y}^2}{\mathrm{X}^2+\mathrm{Y}^2+\mathrm{Z}^2}$	$=\frac{5041}{14021}$
Sin² α_Y	1- Cos² α_Y	$=\frac{14021}{14021}-\frac{5041}{14021}=\frac{9160}{14021}$
Cos α_Z	$\frac{\mathrm{Z}}{\sqrt{\mathrm{X}^2+\mathrm{Y}^2+\mathrm{Z}^2}}$	$=\frac{66}{\sqrt{14021}}$
Cos² α_Z	$\frac{\mathrm{Z}^2}{\mathrm{X}^2+\mathrm{Y}^2+\mathrm{Z}^2}$	$=\frac{4356}{14021}$
Sin² α_Z	1- Cos² α_Z	$=\frac{14021}{14021}=\frac{5041}{14021}=\frac{9160}{14021}$
tan $\beta=\frac{\mathrm{Y}}{\mathrm{X}}=\frac{\mathrm{Cos}\,\alpha_Y}{\mathrm{Cos}\,\alpha_X}$		$=\frac{71}{68}=\frac{\frac{71}{\sqrt{14021}}}{\frac{68}{\sqrt{14021}}}=\frac{71}{68}$
Cos² α_X+ Cos² α_Y+ Cos² α_Z=1		$=\frac{4624}{14021}+\frac{5041}{14021}+\frac{4356}{14021}=\frac{14021}{14021}=1$

Continue practicing 9:

$X = 68$ $\quad$ $o\left|\begin{matrix}0\\0\\0\end{matrix}\right.$

$Y = 71$

$Z = 66$

Cos α_{xy}	$\sqrt{\dfrac{X^2+Y^2}{X^2+Y^2+Z^2}}$	$=\dfrac{\sqrt{4624+5041}}{\sqrt{14021}}=\dfrac{\sqrt{8665}}{\sqrt{14021}}$
Cos² α_{xy}	$\dfrac{X^2+Y^2}{X^2+Y^2+Z^2}$	$=\dfrac{9665}{14021}$
Sin² α_{xy}	1- COS² α_{XY}	$=\dfrac{14021}{14021}-\dfrac{9665}{14021}=\dfrac{4356}{14021}$
Cos α_{xz}	$\sqrt{\dfrac{X^2+Z^2}{X^2+Y^2+Z^2}}$	$=\sqrt{\dfrac{4624+4356}{14021}}=\sqrt{\dfrac{8980}{14021}}$
Cos² α_{xz}	$\dfrac{X^2+Z^2}{X^2+Y^2+Z^2}$	$=\dfrac{8980}{14021}$
Sin² α_{xz}	1- COS² α_{XZ}	$=\dfrac{14021}{14021}-\dfrac{8980}{14021}=\dfrac{5041}{14021}$
Cos α_{yz}	$\sqrt{\dfrac{Y^2+Z^2}{X^2+Y^2+Z^2}}$	$=\sqrt{\dfrac{5041+4356}{14021}}=\sqrt{\dfrac{9397}{14021}}$
Cos² α_{yz}	$\dfrac{Y^2+Z^2}{X^2+Y^2+Z^2}$	$=\dfrac{9397}{14021}$
Sin² α_{yz}	1- COS² α_{YZ}	$=\dfrac{14021}{14021}-\dfrac{9397}{14021}=\dfrac{4624}{14021}$
Cos² α_{xy}+ Cos² α_{xz}+ Cos² α_{yz}=2		$=\dfrac{9665}{14021}+\dfrac{8980}{14021}+\dfrac{9397}{14021}=\dfrac{28042}{14021}=2$
Sin² α_{xy}+ Sin² α_{xz}+ Sin² α_{yz}=1		$=\dfrac{4356}{14021}+\dfrac{5041}{14021}+\dfrac{4624}{14021}=\dfrac{14021}{14021}=1$

tan $\alpha_{xy}=\dfrac{Z}{\sqrt{x^2+y^2}}=\dfrac{66}{\sqrt{9665}}$ $\qquad$ tan $\alpha_{xz}=\dfrac{Y}{\sqrt{x^2+z^2}}=\dfrac{71}{\sqrt{8980}}$

tan $\alpha_{YZ}=\dfrac{X}{\sqrt{y^2+Z^2}}=\dfrac{68}{\sqrt{9397}}$

Practice 10:

$A\left|\begin{matrix}41\\43\\47\end{matrix}\right.\qquad B\left|\begin{matrix}83\\89\\91\end{matrix}\right.$

$$x = \mathrm{X} - \alpha \Rightarrow \mathrm{X} = x - 41 \quad \Rightarrow \quad \mathrm{X} = 83 - 41 = 42$$
$$y = \mathrm{Y} - \beta \Rightarrow \mathrm{Y} = \mathrm{y} - 43 \quad \Rightarrow \quad \mathrm{Y} = 89 - 43 = 46$$
$$z = \mathrm{Z} - \gamma \Rightarrow \mathrm{Z} = \mathrm{z} - 47 \quad \Rightarrow \quad \mathrm{Z} = 91 - 47 = 44$$

Cos α_X	$\frac{\mathrm{X}}{\sqrt{\mathrm{X}^2+\mathrm{Y}^2+\mathrm{Z}^2}}$	$=\frac{42}{\sqrt{1764+2116+1936}}=\frac{42}{\sqrt{5816}}$
Cos² α_X	$\frac{\mathrm{X}^2}{\mathrm{X}^2+\mathrm{Y}^2+\mathrm{Z}^2}$	$=\frac{1764}{5816}$
Sin² α_X	1- Cos² α_X	$=\frac{5816}{5816}-\frac{1764}{5816}=\frac{4052}{5816}$
Cos α_Y	$\frac{\mathrm{Y}}{\sqrt{\mathrm{X}^2+\mathrm{Y}^2+\mathrm{Z}^2}}$	$=\frac{46}{\sqrt{5816}}$
Cos² α_Y	$\frac{\mathrm{Y}^2}{\mathrm{X}^2+\mathrm{Y}^2+\mathrm{Z}^2}$	$=\frac{2116}{5816}$
Sin² α_Y	1- Cos² α_Y	$=\frac{5816}{5816}-\frac{2116}{5816}=\frac{3700}{5816}$
Cos α_Z	$\frac{\mathrm{Z}}{\sqrt{\mathrm{X}^2+\mathrm{Y}^2+\mathrm{Z}^2}}$	$=\frac{44}{\sqrt{5816}}$
Cos² α_Z	$\frac{\mathrm{Z}^2}{\mathrm{X}^2+\mathrm{Y}^2+\mathrm{Z}^2}$	$=\frac{1936}{5816}$
Sin² α_Z	1- Cos² α_Z	$=\frac{5816}{5816}=\frac{1936}{5816}=\frac{3880}{5816}$
tan $\beta=\frac{\mathrm{Y}}{\mathrm{X}}=\frac{\mathrm{Cos}\,\alpha_\mathrm{Y}}{\mathrm{Cos}\,\alpha_\mathrm{X}}$		$=\frac{46}{42}=\frac{\frac{46}{\sqrt{5816}}}{\frac{42}{\sqrt{5816}}}=\frac{46}{42}$
Cos² α_X+ Cos² α_Y+ Cos² α_Z=1		$=\frac{1764}{5816}+\frac{2116}{5816}+\frac{1936}{5816}=\frac{5816}{5816}=1$

Continue practicing 10:

$$X = 42 \qquad Y = 46 \qquad Z = 44 \qquad o\left|\begin{matrix}0\\0\\0\end{matrix}\right.$$

Cos α_{xy}	$\sqrt{\dfrac{X^2+Y^2}{X^2+Y^2+Z^2}}$	$=\dfrac{\sqrt{1764+2116}}{\sqrt{5816}}=\dfrac{\sqrt{3880}}{\sqrt{5816}}$
Cos² α_{xy}	$\dfrac{X^2+Y^2}{X^2+Y^2+Z^2}$	$=\dfrac{3880}{5816}$
Sin² α_{xy}	1- COS² α_{XY}	$=\dfrac{5816}{5816}-\dfrac{3880}{5816}=\dfrac{1936}{5816}$
Cos α_{xz}	$\sqrt{\dfrac{X^2+Z^2}{X^2+Y^2+Z^2}}$	$=\sqrt{\dfrac{1764+1936}{5816}}=\sqrt{\dfrac{3700}{5816}}$
Cos² α_{xz}	$\dfrac{X^2+Z^2}{X^2+Y^2+Z^2}$	$=\dfrac{3700}{5816}$
Sin² α_{xz}	1- COS² α_{XZ}	$=\dfrac{5816}{5816}-\dfrac{3700}{5816}=\dfrac{2116}{5816}$
Cos α_{yz}	$\sqrt{\dfrac{Y^2+Z^2}{X^2+Y^2+Z^2}}$	$=\sqrt{\dfrac{2116+1936}{5816}}=\sqrt{\dfrac{4052}{5816}}$
Cos² α_{yz}	$\dfrac{Y^2+Z^2}{X^2+Y^2+Z^2}$	$=\dfrac{4052}{5816}$
Sin² α_{yz}	1- COS² α_{YZ}	$=\dfrac{5816}{5816}-\dfrac{4052}{5816}=\dfrac{1764}{5816}$
Cos² α_{xy}+ Cos² α_{xz}+ Cos² α_{yz}=2		$=\dfrac{3880}{5816}+\dfrac{3700}{5816}+\dfrac{4052}{5816}=\dfrac{11632}{5816}=2$
Sin² α_{xy}+ Sin² α_{xz}+ Sin² α_{yz}=1		$=\dfrac{1936}{5816}+\dfrac{2116}{5816}+\dfrac{1764}{5816}=\dfrac{5816}{5816}=1$
tan $\alpha_{xy}=\dfrac{Z}{\sqrt{x^2+y^2}}=\dfrac{44}{\sqrt{3880}}$ tan $\alpha_{YZ}=\dfrac{X}{\sqrt{y^2+Z^2}}=\dfrac{42}{\sqrt{4052}}$		tan $\alpha_{xz}=\dfrac{Y}{\sqrt{x^2+z^2}}=\dfrac{46}{\sqrt{3700}}$

Practice 11:

$$A\left|\begin{matrix}49\\51\\53\end{matrix}\right. \qquad B\left|\begin{matrix}73\\79\\83\end{matrix}\right. \qquad \begin{matrix} x = \mathrm{X} + \alpha \Rightarrow \mathrm{X} = x - 49 & \Rightarrow & \mathrm{X} = 49 \\ y = \mathrm{Y} + \beta \Rightarrow \mathrm{Y} = \mathrm{y} - 51 & \Rightarrow & \mathrm{Y} = 51 \\ z = \mathrm{Z} + \gamma \Rightarrow \mathrm{Z} = \mathrm{z} - 53 & \Rightarrow & \mathrm{Z} = 53 \end{matrix}$$

Cos α_X	$\frac{\mathrm{X}}{\sqrt{\mathrm{X}^2+\mathrm{Y}^2+\mathrm{Z}^2}}$	$=\frac{24}{\sqrt{2260}}$
$\mathrm{Cos}^2\ \alpha_X$	$\frac{\mathrm{X}^2}{\mathrm{X}^2+\mathrm{Y}^2+\mathrm{Z}^2}$	$=\frac{576}{2260}$
$\mathrm{Sin}^2\ \alpha_X$	1- $\mathrm{Cos}^2\ \alpha_X$	$=\frac{2260}{2260}-\frac{576}{2260}=\frac{1684}{2260}$
Cos α_Y	$\frac{\mathrm{Y}}{\sqrt{\mathrm{X}^2+\mathrm{Y}^2+\mathrm{Z}^2}}$	$=\frac{28}{\sqrt{2260}}$
$\mathrm{Cos}^2\ \alpha_Y$	$\frac{\mathrm{Y}^2}{\mathrm{X}^2+\mathrm{Y}^2+\mathrm{Z}^2}$	$=\frac{784}{2260}$
$\mathrm{Sin}^2\ \alpha_Y$	1- $\mathrm{Cos}^2\ \alpha_Y$	$=\frac{2260}{2260}-\frac{784}{2260}=\frac{1476}{2260}$
Cos α_Z	$\frac{\mathrm{Z}}{\sqrt{\mathrm{X}^2+\mathrm{Y}^2+\mathrm{Z}^2}}$	$=\frac{30}{\sqrt{2260}}$
$\mathrm{Cos}^2\ \alpha_Z$	$\frac{\mathrm{Z}^2}{\mathrm{X}^2+\mathrm{Y}^2+\mathrm{Z}^2}$	$=\frac{900}{2260}$
$\mathrm{Sin}^2\ \alpha_Z$	1- $\mathrm{Cos}^2\ \alpha_Z$	$=\frac{2260}{2260}=\frac{900}{2260}=\frac{1360}{2260}$
tan $\beta=\frac{\mathrm{Y}}{\mathrm{X}}=\frac{\mathrm{Cos}\ \alpha_Y}{\mathrm{Cos}\ \alpha_X}$		$=\frac{28}{24}=\frac{\frac{28}{\sqrt{2260}}}{\frac{24}{\sqrt{2260}}}=\frac{28}{24}$
$\mathrm{Cos}^2\ \alpha_X+\mathrm{Cos}^2\ \alpha_Y+\mathrm{Cos}^2\ \alpha_Z=1$		$=\frac{576}{2260}+\frac{784}{2260}+\frac{900}{2260}=\frac{2260}{2260}=1$

Continue practicing 11:

$$X = 73 - 49 = 24 \qquad \quad \left| \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right.$$
$$Y = 79 - 51 = 28 \qquad o$$
$$Z = 83 - 53 = 30$$

Cos α_{xy}	$\sqrt{\frac{X^2+Y^2}{X^2+Y^2+Z^2}}$	$=\frac{\sqrt{576+784}}{\sqrt{2260}}=\frac{\sqrt{1360}}{\sqrt{2260}}$
Cos² α_{xy}	$\frac{X^2+Y^2}{X^2+Y^2+Z^2}$	$=\frac{1360}{2260}$
Sin² α_{xy}	1- COS² α_{XY}	$=\frac{2260}{2260}-\frac{1360}{2260}=\frac{900}{2260}$
Cos α_{xz}	$\sqrt{\frac{X^2+Z^2}{X^2+Y^2+Z^2}}$	$=\sqrt{\frac{576+900}{2260}}=\sqrt{\frac{1476}{2260}}$
Cos² α_{xz}	$\frac{X^2+Z^2}{X^2+Y^2+Z^2}$	$=\frac{1476}{2260}$
Sin² α_{xz}	1- COS² α_{XZ}	$=\frac{2260}{2260}-\frac{1476}{2260}=\frac{784}{2260}$
Cos α_{yz}	$\sqrt{\frac{Y^2+Z^2}{X^2+Y^2+Z^2}}$	$=\sqrt{\frac{784+900}{2260}}=\sqrt{\frac{1684}{2260}}$
Cos² α_{yz}	$\frac{Y^2+Z^2}{X^2+Y^2+Z^2}$	$=\frac{1684}{2260}$
Sin² α_{yz}	1- COS² α_{YZ}	$=\frac{2260}{2260}-\frac{1684}{2260}=\frac{576}{2260}$
Cos² α_{xy}+ Cos² α_{xz}+ Cos² α_{yz}=2		$=\frac{1360}{2260}+\frac{1476}{2260}+\frac{1684}{2260}=\frac{4520}{2260}=2$
Sin² α_{xy}+ Sin² α_{xz}+ Sin² α_{yz}=1		$=\frac{900}{2260}+\frac{784}{2260}+\frac{576}{2260}=\frac{2260}{2260}=1$
tan α_{xy}=$\frac{Z}{\sqrt{x^2+y^2}}=\frac{30}{\sqrt{1360}}$ tan α_{YZ}=$\frac{X}{\sqrt{y^2+Z^2}}=\frac{24}{\sqrt{1684}}$		tan α_{xz}=$\frac{Y}{\sqrt{x^2+z^2}}=\frac{28}{\sqrt{1476}}$

Practice 12:

$$A\left|\begin{matrix}53\\57\\59\end{matrix}\right. \qquad B\left|\begin{matrix}63\\67\\71\end{matrix}\right.$$

$$x = \mathrm{X} + \alpha \Rightarrow \mathrm{X} = x - 53 \quad \Rightarrow \quad \mathrm{X} = 63 - 41 = 10$$
$$y = \mathrm{Y} + \beta \Rightarrow \mathrm{Y} = \mathrm{y} - 57 \quad \Rightarrow \quad \mathrm{Y} = 67 - 57 = 10$$
$$z = \mathrm{Z} + \gamma \Rightarrow \mathrm{Z} = \mathrm{z} - 59 \quad \Rightarrow \quad \mathrm{Z} = 71 - 59 = 12$$

Cos α_X	$\frac{\mathrm{X}}{\sqrt{\mathrm{X}^2+\mathrm{Y}^2+\mathrm{Z}^2}}$	$=\frac{10}{\sqrt{100+100+144}} = \frac{10}{\sqrt{344}}$
Cos² α_X	$\frac{\mathrm{X}^2}{\mathrm{X}^2+\mathrm{Y}^2+\mathrm{Z}^2}$	$=\frac{100}{344}$
Sin² α_X	1- Cos² α_X	$=\frac{344}{344}-\frac{100}{344}=\frac{244}{344}$
Cos α_Y	$\frac{\mathrm{Y}}{\sqrt{\mathrm{X}^2+\mathrm{Y}^2+\mathrm{Z}^2}}$	$=\frac{10}{\sqrt{344}}$
Cos² α_Y	$\frac{\mathrm{Y}^2}{\mathrm{X}^2+\mathrm{Y}^2+\mathrm{Z}^2}$	$=\frac{100}{344}$
Sin² α_Y	1- Cos² α_Y	$=\frac{344}{344}-\frac{100}{344}=\frac{244}{344}$
Cos α_Z	$\frac{\mathrm{Z}}{\sqrt{\mathrm{X}^2+\mathrm{Y}^2+\mathrm{Z}^2}}$	$=\frac{12}{\sqrt{344}}$
Cos² α_Z	$\frac{\mathrm{Z}^2}{\mathrm{X}^2+\mathrm{Y}^2+\mathrm{Z}^2}$	$=\frac{144}{344}$
Sin² α_Z	1- Cos² α_Z	$=\frac{344}{344}=\frac{144}{344}=\frac{200}{344}$
tan β=$\frac{\mathrm{Y}}{\mathrm{X}} = \frac{\mathrm{Cos}\,\alpha_{\mathrm{Y}}}{\mathrm{Cos}\,\alpha_{\mathrm{X}}}$		$=\frac{10}{10}=\frac{\frac{10}{\sqrt{344}}}{\frac{10}{\sqrt{344}}}=1$
Cos² α_X+ Cos² α_Y+ Cos² α_Z=1		$=\frac{100}{344}+\frac{100}{344}+\frac{144}{344}=\frac{344}{344}=1$

Continue practicing 12:

$$X = 10 \qquad Y = 10 \qquad Z = 12 \qquad o\left|\begin{matrix}0\\0\\0\end{matrix}\right.$$

Cos α_{xy}	$\sqrt{\dfrac{X^2+Y^2}{X^2+Y^2+Z^2}}$	$=\dfrac{\sqrt{100+100}}{\sqrt{344}}=\dfrac{\sqrt{200}}{\sqrt{344}}$
Cos² α_{xy}	$\dfrac{X^2+Y^2}{X^2+Y^2+Z^2}$	$=\dfrac{200}{344}$
Sin² α_{xy}	1- COS² α_{XY}	$=\dfrac{344}{344}-\dfrac{200}{344}=\dfrac{144}{344}$
Cos α_{xz}	$\sqrt{\dfrac{X^2+Z^2}{X^2+Y^2+Z^2}}$	$=\sqrt{\dfrac{100+144}{344}}=\sqrt{\dfrac{244}{344}}$
Cos² α_{xz}	$\dfrac{X^2+Z^2}{X^2+Y^2+Z^2}$	$=\dfrac{244}{344}$
Sin² α_{xz}	1- COS² α_{XZ}	$=\dfrac{344}{344}-\dfrac{244}{344}=\dfrac{100}{344}$
Cos α_{yz}	$\sqrt{\dfrac{Y^2+Z^2}{X^2+Y^2+Z^2}}$	$=\sqrt{\dfrac{100+144}{344}}=\sqrt{\dfrac{244}{344}}$
Cos² α_{yz}	$\dfrac{Y^2+Z^2}{X^2+Y^2+Z^2}$	$=\dfrac{244}{344}$
Sin² α_{yz}	1- COS² α_{YZ}	$=\dfrac{344}{344}-\dfrac{244}{344}=\dfrac{100}{344}$
Cos² α_{xy}+ Cos² α_{xz}+ Cos² α_{yz}=2		$=\dfrac{200}{344}+\dfrac{244}{344}+\dfrac{244}{344}=\dfrac{688}{344}=2$
Sin² α_{xy}+ Sin² α_{xz}+ Sin² α_{yz}=1		$=\dfrac{144}{344}+\dfrac{100}{344}+\dfrac{100}{344}=\dfrac{344}{344}=1$
tan α_{xy}=$\dfrac{Z}{\sqrt{x^2+y^2}}=\dfrac{12}{\sqrt{200}}$ tan α_{YZ}=$\dfrac{X}{\sqrt{y^2+Z^2}}=\dfrac{10}{\sqrt{244}}$		tan α_{xz}=$\dfrac{Y}{\sqrt{x^2+z^2}}=\dfrac{10}{\sqrt{244}}$

Chapter III

Angular velocity

Calculating angular velocity can be measured in three ways:

A: Prove the angular velocity between two points on any curve. In this method, the proof of angular velocity formula has been proved under the heading of angular velocity at www.p3m.ir and is as follows:

$$\tan \omega = \frac{V_{yA}.V_{xB} - V_{xA}.V_{yB}}{V_{xA}.V_{xB} + V_{yA}.V_{yB}}$$

B: Proving the average angular velocity between several points on the curve: In this method, the formula for the average angular velocity between several points on the curve has been proved on the site www.p3m.ir.

C: Acceleration of instantaneous angles at a point: In this method, if, according to the definition of angular velocity, we obtain the linear velocity distribution on a curve at a point on the radius of rotation$\left(\omega = \frac{V}{R}\right)$, ie in Oxy page (this method of angular velocity is taught graphically in the specialized unit of mechanical engineering under the title of "machine theory" course). Linear Accelerator

(V) is the linear velocity on the curve $V = \sqrt{V^2x + V^2y}$

And (R) has been proven in the topic of sustainable dynamics (fifth dimension) on the site www.p3m.ir. The value (R) is equal to $\overline{oH}$

$$\overline{oH} = R \quad , \quad \overline{oH} = \frac{x \cdot Vy \pm y \cdot Vx}{\sqrt{\boldsymbol{V^2x + V^2y}}}$$

$$\text{Ⓞ}\ \boldsymbol{\omega = \frac{V}{R} = \frac{\sqrt{V^2x+V^2y}}{x\cdot Vy \pm y\cdot Vx \Big/ \sqrt{V^2x+V^2y}} \Rightarrow \frac{V^2x+V^2y}{x\cdot Vy \pm y\cdot Vx} = \omega}$$

The instantaneous angle velocity on each curve is equal to ⓞ

To match the velocity of the instantaneous angle (ω) with $\left(\omega = \frac{d\beta}{d\tau}\right)$on the coordinate plane (oxy) we will have. We have a flat triangle : $\frac{y}{x} = \tan \beta$

$\tan \beta = \frac{y}{x} \Rightarrow (1 + tan^2\beta) \cdot d\beta = \frac{y\cdot dx - x\cdot dy}{x^2}$ We get a differential relationship.

Divide the sides of the relationship by$(\mathrm{d}\tau)$

$$(1+tan^2\beta)\cdot\frac{d\beta}{d\tau}=\frac{-y\cdot {dx}/{d\tau}+x\cdot {dy}/{d\tau}}{x^2}=\frac{-y\cdot Vx+x\cdot Vy}{x^2}$$

$$(1+tan^2\beta)=1+\cdot\frac{y^2}{x^2}=\frac{x^2+y^2}{x^2}\Rightarrow\left(\frac{x^2+y^2}{x^2}\right)\cdot\frac{d\beta}{d\tau}=\frac{-y\cdot Vx+x\cdot Vy}{x^2}$$

$$\text{ⓟ}\ \boldsymbol{\frac{d\beta}{d\tau}=\frac{-y\cdot Vx+x\cdot Vy}{x^2+y^2}}$$

Now the two formulas ⓟ and. Are the instantaneous angular velocities at a point on the curve on the coordinate plane (oxy). Therefore, the two formulas must be equal to each other.

$$\text{ⓞ}\ \boldsymbol{\frac{V^2x+V^2y}{x\cdot Vy\pm y\cdot Vx}=\frac{-y\cdot Vx+x\cdot Vy}{x^2}}\ \text{Ⓟ}$$

Negative option ⓠ⇨ y^2 - x^2= 2c

Positive option ⓡ⇨ $\frac{dy}{dx}=i\sqrt{2+\frac{x^2}{y^2}}$

ⓢ **y=ί. x**

ⓣ $\frac{dy}{dx}=\boldsymbol{i}$

Formula 1 is a hyperbolic equation that is formed in nature as a curve in space. This formation is formed after the Big Bang point explodes, and if a star explodes after the end of its life, its shape will disappear based on the hyperbolic formula. Each galaxy or cluster cloud is a form of motion in its form ه (hyperbolic), and that galaxy or cluster cloud, etc., from the Big Bang point to the present, has continued to rotate safely and without collapse, except for an explosion in the galaxy and ... Allow the hyperbolic shape of the shape to be distorted.
The formula. Applies to some differential equations, and this formula can be considered one of the roots of differential equations.